ANALYSIS OF FOLDED PLATE STRUCTURES BY THE CARRY-OVER METHOD

By

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BY THE CARRY-OVER METHOD

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

NOMENCLATURE

a_x	Longitudinal Distance from Support along Plate
b_j	Width of Plate ij
d_j	Thickness of Plate ij
g_{ij}	Carry-Over Value from m_i to m_j
g'_{ij}	Carry-Over Value from v_i to m_j
g''_{ij}	Carry-Over Value from T_i to m_j
h_j	Horizontal Projection of Plate ij
ℓ_j	Vertical Projection of Plate ij
m_j	Transverse Moment at Joint j
m_{tj}	Transverse Twisting Moment of Beam ij
m_j^*	Starting Value for m_j
p_j	Load Intensity per Unit Plate Area
q_{ij}	Carry-Over Value from T_i to T_j
q'_{ij}	Carry-Over Value from m_i to T_j
q''_{ij}	Carry-Over Value from v_i to T_j
r_{ij}	Carry-Over Value from v_i to v_j
r'_{ij}	Carry-Over Value from m_i to v_j
r''_{ij}	Carry-Over Value from T_i to v_j

t^* Starting Value for T_j

v_j^* Starting Value for v_j

u Displacement in the Longitudinal Direction

v Displacement in the Circumferential Direction

w Displacement Normal to the Plate Surface

x Longitudinal Coordinate

y Transverse Coordinate

z Coordinate Perpendicular to Plate Surface

A_j Transverse Cross-Sectional Area of Plate ij

B_j Width of Folded Plate Structure

E_j Modulus of Elasticity of Plate ij

F_{ji} Angular Flexibility of Plate ij at j

G_{ij} Angular Influence of m_i on m_j

G'_{ij} Coefficient for Influence of v_i on m_j

G''_{ij} Coefficient for Influence of T_i on m_j

H_j Horizontal Thrust Due to $m_j = +1$

H'_j Horizontal Thrust Due to $w_j = +1$

I_j Transverse Moment of Inertia of Plate ij

I'_j Longitudinal Moment of Inertia of Plate ij

J_j Torsional Constant of Plate ij

K_j Plate Constant of Plate ij

L Longitudinal Length of Folded Plate Structure

$M_{jx=a}$	Longitudinal Moment in Plate ij at x=a
$N_{jx=a}$	Normal Force in Plate ij at x=a
P_j	Ridge Load at Joint j
Q_{ij}	Coefficient for Influence of T_i on T_j
Q'_{ij}	Coefficient for Influence of m_i on T_j
Q''_{ij}	Coefficient for Influence of v_i on T_j
R_{ij}	Coefficient for Influence of v_i on v_j
R'_{ij}	Coefficient for Influence of m_i on v_j
R''_{ij}	Coefficient for Influence of T_i on v_j
S_{ij}	Coplanar Plate Load from Ridge Load at i in Plate ij
S_j	Resultant Coplanar Plate Load in Plate ij
S'_j	Resultant Membrane Coplanar Plate Load in Plate ij
ΣT^*_j	Coefficient for Influence of Loads on T_j
ΣV^*	Coefficient for Influence of Loads on v_j
α	$\frac{m\pi}{L}$, m = 1, 2, 3 . . . .
ϕ_j	Angle of Plate ij and the Horizontal
γ_j	Angle Difference Between Plate ij and jk
$\epsilon^{(N)}_{ji}$	Linear Strain at Joint j of Plate ij due to N_j
$\epsilon^{(M)}_{ji}$	Linear Strain at Joint j of Plate ij due to M_j
τ_{ji}	Slope at j of Plate ij Due to Loads

ψ_{ji} Slope at j of Plate ij Due to m_i and m_j

θ_j Slope of Plate ij Due to Displacements

μ Poisson Ratio

ζ, ζ', X Plate Constants

CHAPTER I

INTRODUCTION

1-1 Historical

Papers on the design theory of folded plates began appearing in the 1930's, including the more significant papers by Craemer (1) and Ehlers (2). The first design theories were based on the assumption that the longitudinal joints, ridges, do not undergo displacements and included the derivation of compatible stresses at the junction of plates. This resulted in a series of linear simultaneous equations.

Winter and Pei (3) greatly simplified the solution of the simultaneous equations by recognizing the similarity to the Three Moment Theorem and applied the Cross Distribution Method to the solution of the force equations. This method was later refined by a similar method and the stresses were found directly (4, 5).

Theoretical papers (6, 7) as well as experiments (4) have shown that the assumption of rigid supports at the longitudinal joints results in a large error in many cases. The ridge displacement can not only cause an appreciable change in the transverse moments but also the longitudinal stresses.

The first bending theories (8, 9) developed simultaneous differential equations of the fourth order for determining the ridge displacements. The equations became so complex and their solution so time consuming that most authors chose the displacements as an additional set of variables, then expressed the transverse moments proportional to the second derivative and the external load as a function of the fourth derivative of the displacement.

Vlassow (7) greatly simplified the bending theory by expressing the ridge displacements as a function of the longitudinal stresses and the transverse moments and loads.

1-2 General

Folded plate structures are generally categorized into three groups: large, medium, and small length-to-height ratios. A different theory is used to analyze each of the three groups. This thesis is concerned with the medium length-to-height ratios in which the compatibility of plate displacements, longitudinal shears, and transverse moments is required for an accurate analysis.

The three types of deformation equations necessary to solve the folded plate structure are presented. The Shear Flow Equation is formulated by equating the longitudinal strains at a longitudinal joint. By summing the transverse angle changes at a joint to zero, the Three Moment Equation is obtained. From the general beam equation, the

Plate Displacement Equation is written in terms of the transverse loads, moments, and longitudinal shears.

The main purpose of this thesis is to extend the carry-over method of solution to the folded plate deformation equations. The carry-over procedure is a systematic process by which a set of simultaneous equations can be solved. This can be accomplished by a one table carry-over with three sets of unknowns or a three table carry-over, each with a set of unknowns.

To reduce the number of simultaneous equations, the displacement functions are substituted into the Three Moment Equation yielding the Seven Moment Equations. Thus, a one table carry-over with two sets of unknowns or a two table carry-over, each with a set of unknowns, can be utilized.

Particular deformation equations for folded plate structures with vertical edge members are included. The plate theory is utilized for thickened edge plates which are assumed to resist bending about the y and z axes and also twisting. The beam theory is utilized for edge members with beam depth-to-thickness ratios and are assumed to resist bending about the x and y axes and torsion about the x axis.

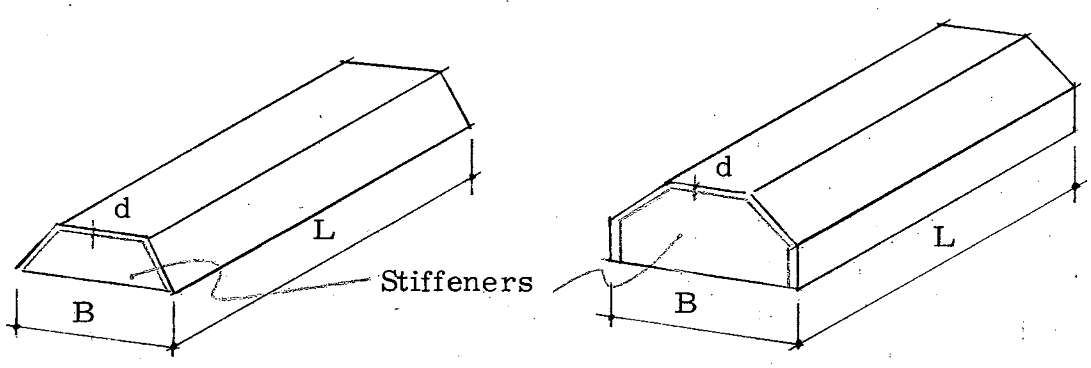
CHAPTER II

GENERAL THEORY

2-1 Basic Assumptions

The nomenclature used in this thesis is that used by Girkmann (10) with exceptions necessary to conform to previous nomenclature established at Oklahoma State University.

The folded plate structure, sometimes called "hipped plate" or "prismatic shell", consists of two or more thin planar elements called plates, supported by two or more transverse stiffeners. The length L is the dimension between transverse stiffeners, the width B is the dimension between longitudinal edges, and the thickness d is the perpendicular dimension between surfaces (Fig. 2-1).



(a)
Folded Plate Without
Edge Beams

(b)
Folded Plate With
Edge Beams

Fig. 2-1
Typical Folded Plates

The following assumptions are those most commonly adopted in the analysis of folded plate structures and will be assumed valid in this presentation.

a) The material of the plate is homogeneous, isotropic, and continuous.

b) Stresses developed are below the elastic limit.

c) A plane surface before deformation remains plane after deformation.

d) The material of the plate is elastic and follows Hooke's Law.

e) Twisting resistance of the individual interior plates is neglected and acts only in the heavier edge plates.

f) The principle of superposition holds true.

g) Longitudinal plate action is neglected for intermediate and large length-to-height ratios.

h) The supporting transverse stiffeners are assumed incapable of providing resistance against rotation of the ends of the plates in their own plane but assumed capable of providing infinite resistance against transverse rotation or translation.

i) Longitudinal joints are continuous such that joint rotations and displacements are common.

j) Where the length-to-height ratio is small, the variation in the transverse moments due to displacements is small and can be neglected.

k) For intermediate ratios of length-to-height, the displacements are significant and should be accounted for as well as the compatibility of transverse moments and longitudinal shears.

l) Where the length-to-height ratio is large and there is a series of folded plate repetitions, the stress variation is closely approximated by assuming the transverse section as a beam section.

2-2 Sign Convention

The following sign convention is adopted:

a) Loads are assumed positive if acting vertical and down (Fig. 2-2a). Any inclined or horizontal load requires a force polygon other than that shown in Fig. 2-4.

b) Transverse Plate Loads are assumed positive if acting from right to left (Fig. 2-2b).

c) Transverse Bending Moments are assumed positive if acting as shown in Fig. 2-2c).

d) Plate Displacements are assumed positive if acting from right to left (Fig. 2-2d).

e) The angles measuring the geometry of the plates receive their quadrant sign and are measured as shown in Fig. 2-2e.

f) Longitudinal shear is assumed positive if acting as shown in Fig. 2-2f.

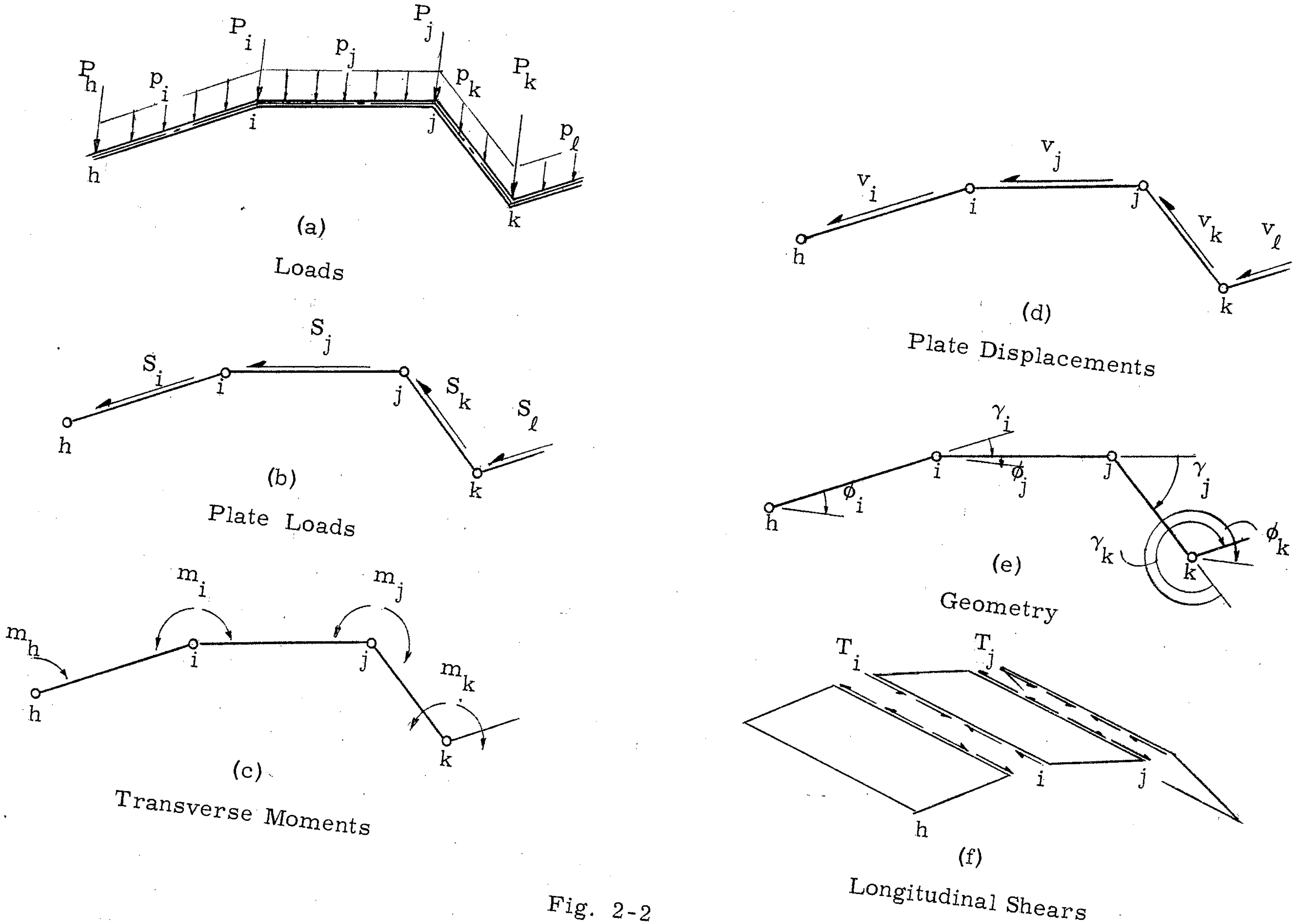


Fig. 2-2
Sign Convention

2-3 Basic Folded Plate Theory

A typical transverse section hijk is considered (Fig. 2-3a). The uniform load p causes transverse bending moments occurring between $x = 0$ and $x = L$. Assuming in Fig. 2-3b the longitudinal joints to be supported, the transverse moments developed are analogous to the moments of a continuous beam.

Since the interior plates are assumed to have no resistance to twisting or longitudinal plate action, the end reactions of the analogous transverse beam can be resisted only by coplanar plate loads S (Fig. 2-3c). The total end reaction or joint loads P are

$$\begin{aligned} P_j &= p_j \frac{b_j}{2} + p_k \frac{b_k}{2} \\ P_k &= p_k \frac{b_k}{2} + p_\ell \frac{b_\ell}{2} \end{aligned} \tag{2-1}$$

and from a force polygon at each joint (Fig. 2-4), the coplanar plate loads are

$$\begin{aligned} S_{ij} &= P_i \frac{\cos\phi_i}{\sin\gamma_i} \\ S_{ji} &= P_j \frac{\cos\phi_k}{\sin\gamma_j} \\ S_{jk} &= P_j \frac{\cos\phi_j}{\sin\gamma_j} \\ S_{kj} &= P_k \frac{\cos\phi_\ell}{\sin\gamma_k} \; . \end{aligned} \tag{2-2}$$

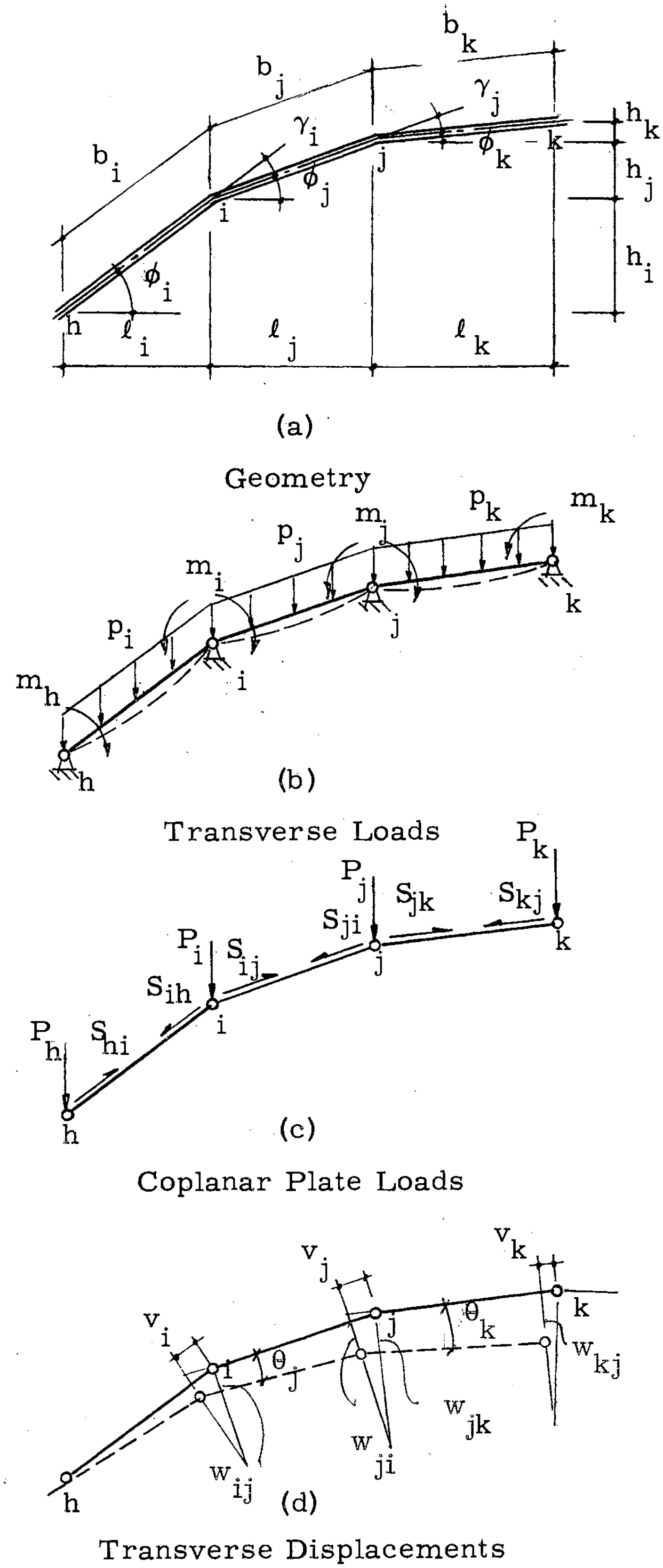
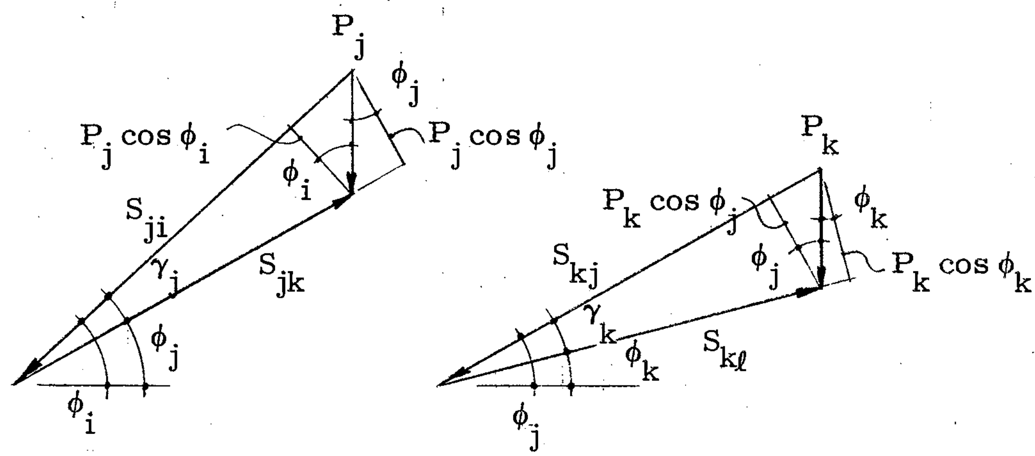


Fig. 2-3

Typical Transverse Section



(a)
Joint j

(b)
Joint k

Fig. 2-4

Typical Force Polygons

The membrane plate loads (loads neglecting variation in transverse moment) are

$$\begin{aligned} S'_j &= S_{ji} - S_{ij} = P_j \frac{\cos\phi_k}{\sin\gamma_j} - P_i \frac{\cos\phi_i}{\sin\gamma_i} \\ S'_k &= S_{kj} - S_{jk} = P_k \frac{\cos\phi_\ell}{\sin\gamma_k} - P_j \frac{\cos\phi_j}{\sin\gamma_j} \ . \end{aligned} \tag{2-3}$$

Due to the coplanar plate loads, transverse plate displacements will occur. From the geometry of the plate, the slope of any typical plate is (Fig. 2-3d)

$$\begin{aligned} \theta_j &= + \frac{1}{b_k} (w_{kj} - w_{jk}) \\ &= \frac{1}{b_k} \left[(v_j \cot\gamma_j - \frac{v_k}{\sin\gamma_j}) - (\frac{v_i}{\sin\gamma_i} - v_j \cot\gamma_i) \right] \end{aligned}$$

or

$$\theta_j = \frac{1}{b_k}\left[v_j(\cot\gamma_j + \cot\gamma_i) - \frac{v_i}{\sin\gamma_i} - \frac{v_k}{\sin\gamma_j}\right] \qquad (2\text{-}4)$$

The change in slope of the individual plates produces a change in the transverse moments and thus a change in the longitudinal joint load (Chapter IV).

Longitudinal shearing forces (Fig. 2-5) are developed between the individual plates as the plates undergo transverse displacements (Chapter III).

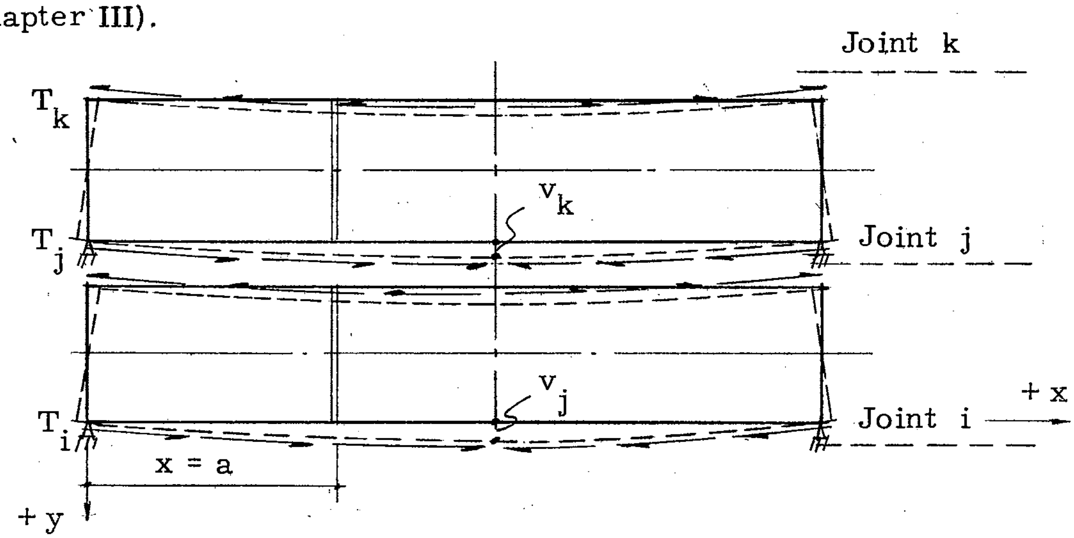


Fig. 2-5

Longitudinal Section

The analysis of the folded plate structure consists in establishing compatibility between all plate deformations. Once the longitudinal shears, transverse moments, and plate displacements are evaluated to satisfy the deformation relationships, the proper solution is attained for the particular loading on the structure.

CHAPTER III

SHEAR FLOW EQUATION

3-1 Statics and Representation of Functions

A typical longitudinal element of an interior plate ij having a thickness d_j and incremental length dx is considered (Fig. 3-1).

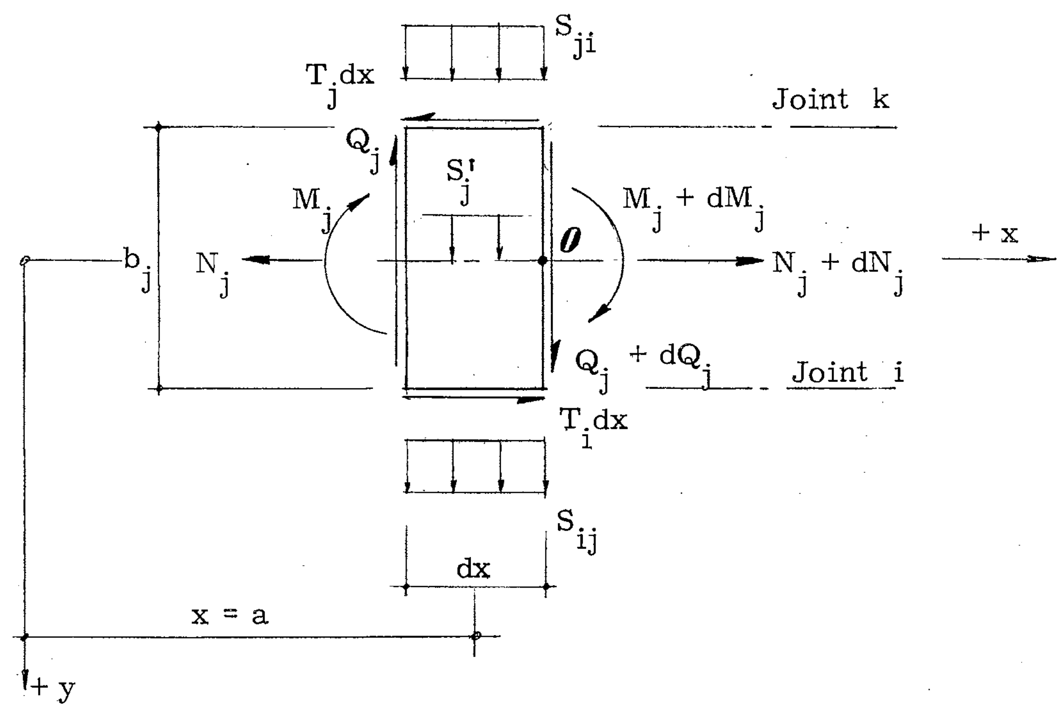


Fig. 3-1

Typical Longitudinal Element

The longitudinal shear flow at joints i and j is denoted as T_i and T_j respectively and the resultant of these shears is denoted as N_j.

The longitudinal bending moment is denoted as M_j and the resultant membrane plate load is denoted S'_j .

Forces are summed in the x and y directions, obtaining Eq's. (3-1, 2).

$$dQ_j = - S'_j \, dx \tag{3-1}$$

$$dN_j = (T_j - T_i) \, dx \, . \tag{3-2}$$

Moments are summed about point O , obtaining Eq. (3-3).

$$dM_j = Q_j dx - \tfrac{1}{2}(T_j - T_i) b_j dx \, . \tag{3-3}$$

The various functions can be represented by a general Fourier Series. If the structure is simply supported, the general Fourier Series representations of the plate functions are

$$\begin{aligned} Q_j &= \Sigma_n \overline{Q}_{jn} \cos \alpha x \\ T_j &= \Sigma_n \overline{T}_{jn} \cos \alpha x \\ S_j &= \Sigma_n \overline{S}_{jn} \sin \alpha x \\ N_j &= \Sigma_n \overline{N}_{jn} \sin \alpha x \\ M_j &= \Sigma_n \overline{M}_{jn} \sin \alpha x \end{aligned} \tag{3-4}$$

where

$$\alpha = \frac{n \pi}{L} \, .$$

Since the principle of superposition holds true, only a typical term of the series needs to be considered.

Integrating Eq's. (3-1, 2, 3) for a typical term of the series and substituting known boundary conditions, Eq's. (3-1, 2, 3) become

$$\overline{Q}_j = \frac{1}{\alpha} \overline{S}_j \tag{3-5a}$$

$$\overline{N}_j = \frac{1}{\alpha} (\overline{T}_j - \overline{T}_i) \tag{3-5b}$$

$$\overline{M}_j = \frac{1}{\alpha} \overline{Q}_j - \frac{b_j}{2\alpha} (\overline{T}_j + \overline{T}_i). \tag{3-5c}$$

3-2 Longitudinal Deformation Equation

The unit strains along the longitudinal joint j in plates ij and jk respectively are

$$\begin{aligned} \epsilon_{ji} &= \epsilon_{ji}^{(N)} + \epsilon_{ji}^{(M)} = \frac{N_j}{A_j E_j} - \frac{M_j \frac{b_j}{2}}{I_j E_j} \\ \epsilon_{jk} &= \epsilon_{jk}^{(N)} + \epsilon_{jk}^{(M)} = \frac{N_k}{A_k E_k} + \frac{M_k \frac{b_k}{2}}{I_k E_k} \end{aligned} \tag{3-6}$$

where

$\epsilon^{(N)} =$ strain due to normal force

$\epsilon^{(M)} =$ strain due to longitudinal bending moment

and

$$I_j = \frac{b_j d_j^3}{12}, \qquad A_j = b_j d_j$$

$$I_k = \frac{b_k d_k^3}{12}, \qquad A_k = b_k d_k .$$

Since the plates are rigidly connected along the longitudinal joint j, the longitudinal strains must be equal (Fig. 3-2).

$$\epsilon_{ji} = \epsilon_{jk}.$$

Substituting the expressions for the unit strains from Eq's. (3-6)

$$\frac{N_j}{A_j E_j} - \frac{M_j b_j}{2 I_j E_j} = \frac{N_k}{A_k E_k} + \frac{M_k b_k}{2 I_k E_k} . \qquad (3\text{-}7)$$

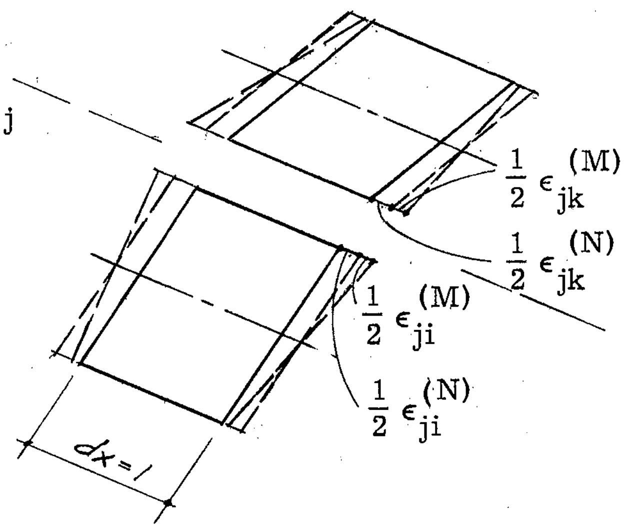


Fig. 3-2

Longitudinal Strains at Joint j

Substituting the values of Eq. (3-5)

$$\frac{1}{\alpha A_j E_j}(\overline{T}_j - \overline{T}_i) - \frac{b_j}{2 I_j E_j}\left[\frac{1}{\alpha}\overline{Q}_j - \frac{b_j}{2\alpha}(\overline{T}_j + \overline{T}_i)\right]$$

$$= \frac{1}{\alpha A_k E_k}(\overline{T}_k - \overline{T}_j) + \frac{b_k}{2 E_k I_k}\left[\frac{1}{\alpha}\overline{Q}_k - \frac{b_k}{2\alpha}(\overline{T}_k + \overline{T}_j)\right],$$

and then substituting the values of $\overline{Q}_j$ and $\overline{Q}_k$ the final equation with E a constant is

$$\frac{1}{A_j}\,\overline{T}_i + 2\left(\frac{1}{A_j} + \frac{1}{A_k}\right)\overline{T}_j + \frac{1}{A_k}\,\overline{T}_k = \frac{3}{\alpha}\left(\frac{\overline{S}_j}{A_j b_j} + \frac{\overline{S}_k}{A_k b_k}\right) . \qquad (3\text{-}8)$$

Referring to a transverse section (Fig. 3-3) acted upon by transverse moments $\overline{m}$, the change in the longitudinal joint load P_j due to the transverse moments is

$$\Delta\overline{P}_j = \left(\frac{\overline{m}_i - \overline{m}_j}{\ell_j}\right) + \left(\frac{\overline{m}_k - \overline{m}_j}{\ell_k}\right) . \qquad (3\text{-}9)$$

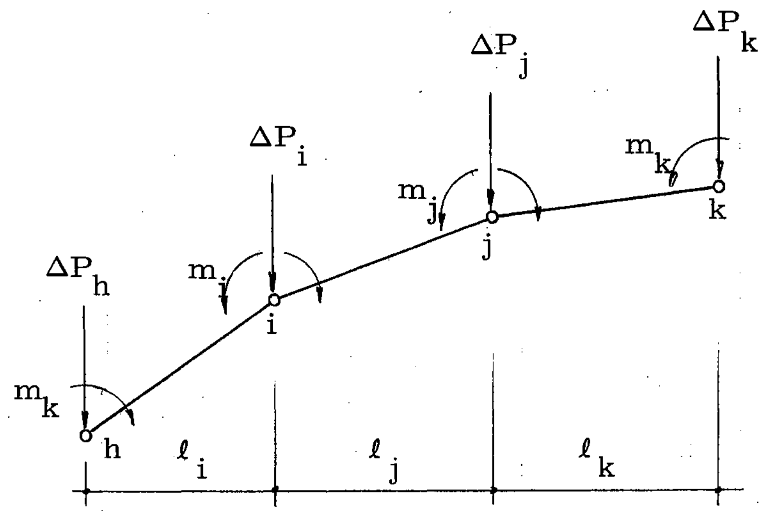


Fig. 3-3

Load Variation from Transverse Moments

The total typical plate load

$$\overline{S}_j = (\overline{S}_{ji} + \Delta\overline{S}_{ji}) - (\overline{S}_{ij} + \Delta\overline{S}_{ij}) \qquad (3\text{-}10)$$

and from Chapter II

$$\begin{aligned} \overline{S}_{ji} &= \overline{P}_j \frac{\cos\phi_k}{\sin\gamma_j} , & \Delta\overline{S}_{ji} &= \Delta\overline{P}_j \frac{\cos\phi_k}{\sin\gamma_j} \\ \overline{S}_{ij} &= \overline{P}_i \frac{\cos\phi_i}{\sin\gamma_i} , & \Delta\overline{S}_{ij} &= \Delta\overline{P}_i \frac{\cos\phi_i}{\sin\gamma_i} . \end{aligned} \qquad (3\text{-}11)$$

The loads can be represented by a general Fourier Series. If the structure is simply supported, the general Fourier Series representations of the loads are

$$p_j = \sum_n \overline{p}_{jn} \sin \alpha x$$

$$P_j = \sum_n \overline{P}_{jn} \sin \alpha x \qquad (3\text{-}12)$$

$$\Delta P_j = \sum_n \Delta \overline{P}_{jn} \sin \alpha x \ .$$

Substituting Eq's. (3-9, 10, 11, 12) into the general shear flow equation (Eq. 3-8), the resulting "Three Shear Flow Equation with Moment Influence" at joint j is

$$Q_{ij}\overline{T}_i + \Sigma Q_j \overline{T}_j + Q_{kj}\overline{T}_k = Q'_{hj}\overline{m}_h + Q'_{ij}\overline{m}_i + Q'_{jj}\overline{m}_j + Q'_{kj}\overline{m}_k$$

$$+ Q'_{\ell j}\overline{m}_\ell + \Sigma \overline{T}^*_j . \qquad (3\text{-}13)$$

Writing in carry-over form, Eq. (3-13) becomes

$$\overline{T}_j = + q_{ij}\overline{T}_i + q_{kj}\overline{T}_k + q'_{hj}\overline{m}_h + q'_{ij}\overline{m}_i + q'_{jj}\overline{m}_j + q'_{kj}\overline{m}_k$$

$$+ q'_{\ell j}\overline{m}_\ell + \overline{t}^*_j . \qquad (3\text{-}14)$$

The values of the coefficients in Eq's. (3-13, 14) are tabulated in the Appendix (Table 1).

CHAPTER IV

DISPLACEMENT AND MOMENT EQUATIONS

4-1 Derivation of the Displacement Equation

A typical transverse section through the structure hijk is considered (Fig. 4-1). Since the structure is simply supported only at the transverse end stiffeners, (x = 0 and x = L) displacements occur at the longitudinal joints between the end stiffeners.

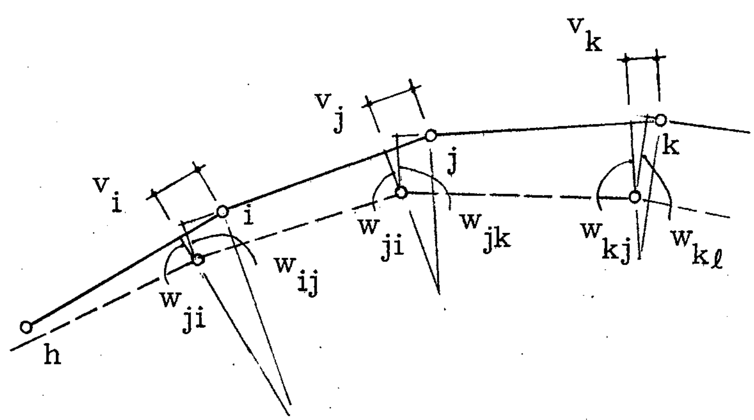


Fig. 4-1
Typical Transverse Section

The displacement v_j of a typical plate ij can be represented by a Fourier Series. A typical term of the series for the displacement of Plate ij is

$$v_j = \bar{v}_j \sin \alpha x \ ,$$

and differentiating twice with respect to x

$$\frac{d^2 v_j}{dx^2} = - \bar{v}_j \alpha^2 \sin \alpha x \quad . \tag{4-1}$$

The differential equation for the longitudinal bending moment is

$$EI_j \frac{d^2 v_j}{dx^2} = - M_j \quad . \tag{4-2}$$

Substitution of Eq's. (4-1, 3-5c) into Eq. (4-2) yields

$$- EI_j \bar{v}_j \alpha^2 \sin \alpha x = - \frac{1}{\alpha} \left[\bar{Q}_j - \frac{b_j}{2} (\bar{T}_j + \bar{T}_i) \right] \sin \alpha x$$

and since

$$\bar{Q}_j = + \frac{1}{\alpha} \bar{S}_j \quad ,$$

the equation becomes

$$EI_j \alpha^4 \bar{v}_j = \bar{S}_j - (\bar{T}_j + \bar{T}_i) \frac{\alpha b_j}{2} \quad . \tag{4-3}$$

The total plate load S_j from Eq. (3-10) is

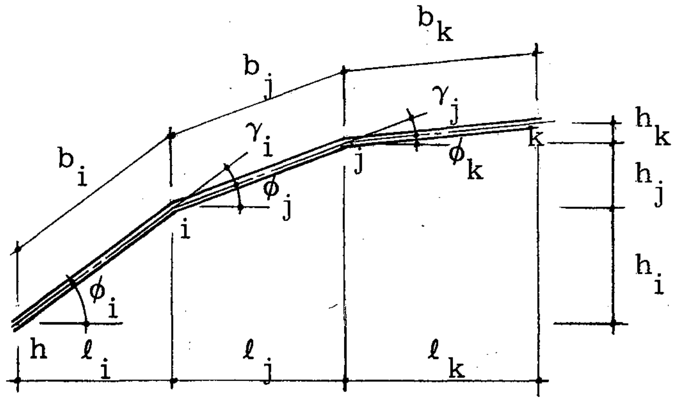
$$\bar{S}_j = (\bar{S}_{ji} + \Delta \bar{S}_{ji}) - (\bar{S}_{ij} + \Delta \bar{S}_{ij}) \quad . \tag{4-4}$$

Substituting the values of S and also the values of ΔS (Eq's. 3-11), the "Displacement Equation with Moment and Shear Influence" is

$$\Sigma R_j \bar{v}_j = + R'_{hj} \bar{m}_h + R'_{ij} \bar{m}_i + R'_{jj} \bar{m}_j$$

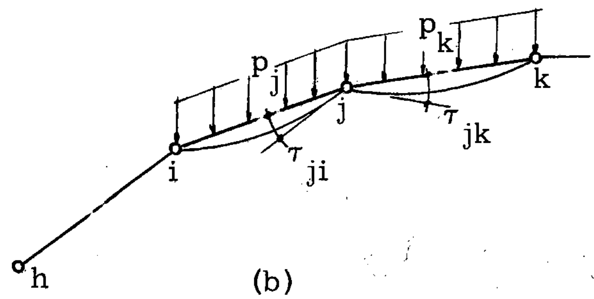
$$+ R'_{kj} \bar{m}_k + R'_{\ell j} \bar{m}_\ell + R''_{ij} \bar{T}_i + R''_{jj} \bar{T}_j + \Sigma \bar{V}^*_j . \tag{4-5}$$

Written in carry-over form, Eq. (4-5) becomes



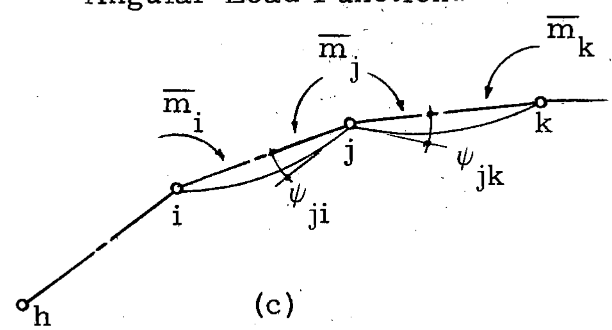
(a)

Geometry



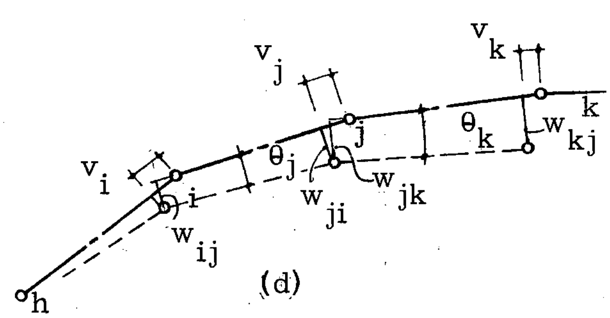
(b)

Angular Load Functions



(c)

Angular Moment Functions



(d)

Angular Displacement Functions

Fig. 4-2

Angular Deformations at Joint j

$$\bar{v}_j = + r'_{hj}\bar{m}_h + r'_{ij}\bar{m}_i + r'_{jj}\bar{m}_j + r'_{kj}\bar{m}_k + r'_{\ell j}\bar{m}_\ell$$

$$+ r''_{ij}\bar{T}_i + r''_{jj}\bar{T}_j + \bar{v}^*_j . \qquad (4\text{-}6)$$

The values of the coefficients in Eq's. (4-5, 6) are tabulated in the Appendix (Table 2).

4-2 Derivation of the Three Moment Equation

The equations for the solution of the redundant transverse moments are obtained by summing the total angular deformation at each joint to zero. A typical joint j is considered with the separate angular deformations shown in Fig. 4-2.

a) Angular Load Function

$$\tau_{jk} = \int_j^k \frac{Bm_k y\,dy}{b_k EI'_k} = \frac{p_k b_k^2 \ell_k}{24EI_k}$$

$$\tau_{ji} = \int_j^i \frac{Bm_j y'\,dy}{b_j EI'_j} = \frac{p_j b_j^2 \ell_j}{24EI'_j}$$

b) Angular Moment Functions

$$\psi_{jk} = F_{jk}m_j + G_{kj}m_k = m_j\int_j^k \frac{y^2 dy}{b_k^2 EI'_k} + m_k\int_j^k \frac{yy'\,dy}{b_k^2 EI'_k}$$

$$= \frac{b_k}{3EI'_k}\, m_j + \frac{b_k}{6EI'_k}\, m_k$$

$$\psi_{ji} = F_{ji}m_j + G_{ij}m_i = m_j\int_j^i \frac{y'^2 dy}{b_j EI'_j} + m_i\int_j^i \frac{yy'dy}{b_j^2 EI'_j}$$

$$= \frac{b_j}{3EI'_j} m_j + \frac{b_j}{6EI'_j} m_i \quad .. \tag{4-8}$$

c) Angular Displacement Functions

$$\theta_j = \frac{1}{b_j} (w_{ji} - w_{ij})$$

$$\theta_k = \frac{1}{b_k} (w_{kj} - w_{jk}) \tag{4-9}$$

where

$$w_{ji} = v_j \cot \gamma_j - \frac{v_k}{\sin \gamma_j}$$

$$w_{ij} = \frac{v_i}{\sin \gamma_i} - v_j \cot \gamma_i$$

$$w_{kj} = v_k \cot \gamma_k - \frac{v_\ell}{\sin \gamma_k}$$

$$w_{jk} = \frac{v_j}{\sin \gamma_j} - v_k \cot \gamma_j \quad .$$

The equation for the sum of angular changes at a typical joint j from Fig. 4-2 is

$$0 = \tau_{ji} + \tau_{jk} + \psi_{ji} + \psi_{jk} + \theta_k - \theta_j \; . \tag{4-10}$$

Substituting Eq's. (4-7, 8, 9) and writing for a typical term of the Fourier Series, Eq. (4-10) becomes

$$0 = \Sigma \overline{\tau}_j + G_{ij}\overline{m}_i + \Sigma F_j \overline{m}_j + G_{kj}\overline{m}_k$$

$$+ \frac{1}{b_k} \left[\overline{v}_k(\cot \gamma_k + \cot \gamma_j) - \frac{\overline{v}_j}{\sin \gamma_j} - \frac{\overline{v}_\ell}{\sin \gamma_k} \right]$$

$$- \frac{1}{b_j}\left[v_j(\cot\gamma_j + \cot\gamma_i) - \frac{\overline{v}_i}{\sin\gamma_i} - \frac{\overline{v}_k}{\sin\gamma_k}\right] \quad . \tag{4-11}$$

Rewriting Eq. (4-11), the "Three Moment Equation with Displacement Influence" is

$$\begin{aligned} \Sigma F_j \overline{m}_j \quad = \quad & - G_{ij}\overline{m}_i - G_{kj}\overline{m}_k - G'_{ij}\overline{v}_i - G_{jj}\overline{v}_j \\ & - G'_{kj}\overline{v}_k - G'_{\ell j}\overline{v}_\ell - \Sigma\overline{\tau}_j \quad . \end{aligned} \tag{4-12}$$

Writing in carry-over form, Eq. (4-12) becomes

$$\begin{aligned} \overline{m}_j \quad = \quad & + g_{ij}\overline{m}_i + g_{kj}\overline{m}_k + g'_{ij}\overline{v}_i + g'_{jj}\overline{v}_j \\ & + g'_{kj}\overline{v}_k + g'_{\ell j}\overline{v}_\ell + \overline{m}^*_j \quad . \end{aligned} \tag{4-13}$$

The values of the coefficients in Eq's. (4-12, 13) are tabulated in the Appendix (Table 3).

4-3 Derivation of the Seven Moment Equation

If the values of v are substituted into Eq. (4-12) from Eq. (4-6), the "Seven Moment Equation with Shear Influence" is

$$\begin{aligned} \Sigma F_j \overline{m}_j \quad = \quad & - G_{gj}\overline{m}_g - G_{hj}\overline{m}_h - G_{ij}\overline{m}_i - G_{kj}\overline{m}_k \\ & - G_{\ell j}\overline{m}_\ell - G_{mj}\overline{m}_m - G''_{hj}\overline{T}_h - G''_{ij}\overline{T}_i \\ & - G''_{jj}\overline{T}_j - G''_{kj}\overline{T}_k - G''_{\ell j}\overline{T}_\ell - \Sigma\overline{\tau}^*_j \quad . \end{aligned} \tag{4-14}$$

Writing in carry-over form, Eq. (4-14) becomes

$$\overline{m}_j = + g_{gj}\overline{m}_g + g_{hj}\overline{m}_h + g_{ij}\overline{m}_i + g_{kj}\overline{m}_k + g_{\ell j}\overline{m}_\ell$$
$$+ g_{mj}\overline{m}_m + g''_{hj}\overline{T}_h + g''_{ij}\overline{T}_i + g''_{jj}\overline{T}_j + g''_{kj}\overline{T}_k$$
$$+ g''_{\ell j}\overline{T}_\ell + \overline{m}_j^* \; . \tag{4-15}$$

The values of the coefficients in Eq's. (4-14, 15) are tabulated in the Appendix (Table 4).

CHAPTER V

SOLUTION BY CARRY-OVER

5-1 One Table Carry-Over

The three sets of deformation equations from Chapters III and IV, combined with the equations from Chapter VI for edge members, comprise the equations necessary to solve the folded plate structure. For n number of plates, there are n-1 shear equations, n displacement equations, and n-1 moment equations. The equations written in matrix form are

$$\begin{bmatrix}
\Sigma Q_i & Q_{ji} & & & & & & & -Q'_{ii} & -Q'_{ji} & -Q'_{ki} & \cdots \\
Q_{ij} & \Sigma Q_j & Q_{kj} & & & & & & -Q'_{ij} & -Q'_{jj} & -Q'_{kj} & \cdots \\
 & Q_{jk} & \Sigma Q_k & \cdots & & & & & -Q'_{ik} & -Q'_{jk} & -Q'_{kk} & \cdots \\
 & & \cdots & \cdots & & & & & & \cdots & \cdots & \cdots \\
-R''_{ii} & & & & \Sigma R_i & & & & -R'_{ii} & -R'_{ji} & \cdots & \\
-R''_{ij} & -R''_{jj} & & & & \Sigma R_j & & & -R'_{ij} & -R'_{jj} & -R'_{kj} & \\
 & -R''_{jk} & -R''_{kk} & & & & \Sigma R_k & & -R'_{ik} & -R'_{jk} & -R'_{kk} & \cdots \\
 & & \cdots & \cdots & & & & \cdots & & \cdots & \cdots & \cdots \\
 & & & & G'_{ii} & G'_{ji} & G'_{kj} & \cdots & \Sigma F_i & G_{ji} & & \\
 & & & & G'_{ij} & G'_{jj} & G'_{kj} & \cdots & G_{ij} & \Sigma F_j & G_{kj} & \\
 & & & & & G'_{jk} & G'_{kk} & \cdots & & G_{jk} & \Sigma F_k & \cdots \\
 & & & & & & \cdots & \cdots & & & \cdots & \cdots
\end{bmatrix}
\begin{bmatrix}
\overline{T}_i \\ \overline{T}_j \\ \overline{T}_k \\ \cdots \\ \overline{v}_i \\ \overline{v}_j \\ \overline{v}_k \\ \cdots \\ \overline{m}_i \\ \overline{m}_j \\ \overline{m}_k \\ \cdots
\end{bmatrix}
=
\begin{bmatrix}
\Sigma\overline{T}^*_i \\ \Sigma\overline{T}^*_j \\ \Sigma\overline{T}^*_k \\ \cdots \\ \Sigma\overline{V}^*_i \\ \Sigma\overline{V}^*_j \\ \Sigma\overline{V}^*_k \\ \cdots \\ -\Sigma\overline{\tau}^*_i \\ -\Sigma\overline{\tau}^*_j \\ -\Sigma\overline{\tau}^*_k \\ \cdots
\end{bmatrix} \qquad (5\text{-}1)$$

Rewriting Eq. (5-1) in a form suitable for a carry-over solution,

$$\begin{bmatrix}
+1 & q_{ji} & & & & & & & -q'_{ii} & -q'_{ji} & -q'_{ki} & \cdot\cdot \\
q_{ij} & +1 & q_{kj} & & & & & & -q'_{ij} & -q'_{jj} & -q'_{kj} & \cdot\cdot \\
 & q_{jk} & +1 & & & & & & -q'_{ik} & -q'_{jk} & -q'_{kk} & \cdot\cdot \\
 & & \cdot\cdot & \cdot\cdot & & & & & & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
-r''_{ii} & & & & +1 & & & & -r'_{ii} & -r'_{ji} & & \\
-r''_{ij} & -r''_{jj} & & & & +1 & & & -r'_{ij} & -r'_{jj} & -r'_{kj} & \cdot\cdot \\
 & -r''_{jk} & -r''_{kk} & & & & +1 & & -r'_{ik} & -r'_{jk} & -r'_{kk} & \cdot\cdot \\
 & & \cdot\cdot & \cdot\cdot & & & & \cdot\cdot & & & \cdot\cdot & \cdot\cdot \\
 & & & & g'_{ii} & g'_{ji} & g'_{ki} & & +1 & g_{ji} & & \\
 & & & & g'_{ij} & g'_{jj} & g'_{kj} & & g_{ij} & +1 & g_{kj} & \\
 & & & & & g'_{jk} & g'_{kk} & & & g_{jk} & +1 & \cdot\cdot \\
 & & & & & & \cdot\cdot & \cdot\cdot & & & \cdot\cdot & \cdot\cdot
\end{bmatrix}
\begin{bmatrix} \bar{T}_i \\ \bar{T}_j \\ \bar{T}_k \\ \cdot\cdot \\ \bar{v}_i \\ \bar{v}_j \\ \bar{v}_k \\ \cdot\cdot \\ \bar{m}_i \\ \bar{m}_j \\ \bar{m}_k \\ \cdot\cdot \end{bmatrix}
=
\begin{bmatrix} \bar{t}^*_i \\ \bar{t}^*_j \\ \bar{t}^*_k \\ \cdot\cdot \\ \bar{v}^*_i \\ \bar{v}^*_j \\ \bar{v}^*_k \\ \cdot\cdot \\ \bar{m}^*_i \\ \bar{m}^*_j \\ \bar{m}^*_k \\ \cdot\cdot \end{bmatrix}
\qquad (5\text{-}2)$$

The solution of the simultaneous equations by a carry-over procedure can conveniently be presented in tabular form (Fig. 5-1). The shear carry-over factors are q (shear to shear) and q' (moment to shear). The displacement carry-over factors are r' (moment to displacement) and r'' (shear to displacement). The moment carry-over factors are g (moment to moment) and g' (displacement to moment). For an edge member, there are additional carry-over factors r (displacement to displacement) which relate the displacements of the edge member and the adjacent plate. The carry-over procedure is a converging series which can be continued until the desired accuracy is

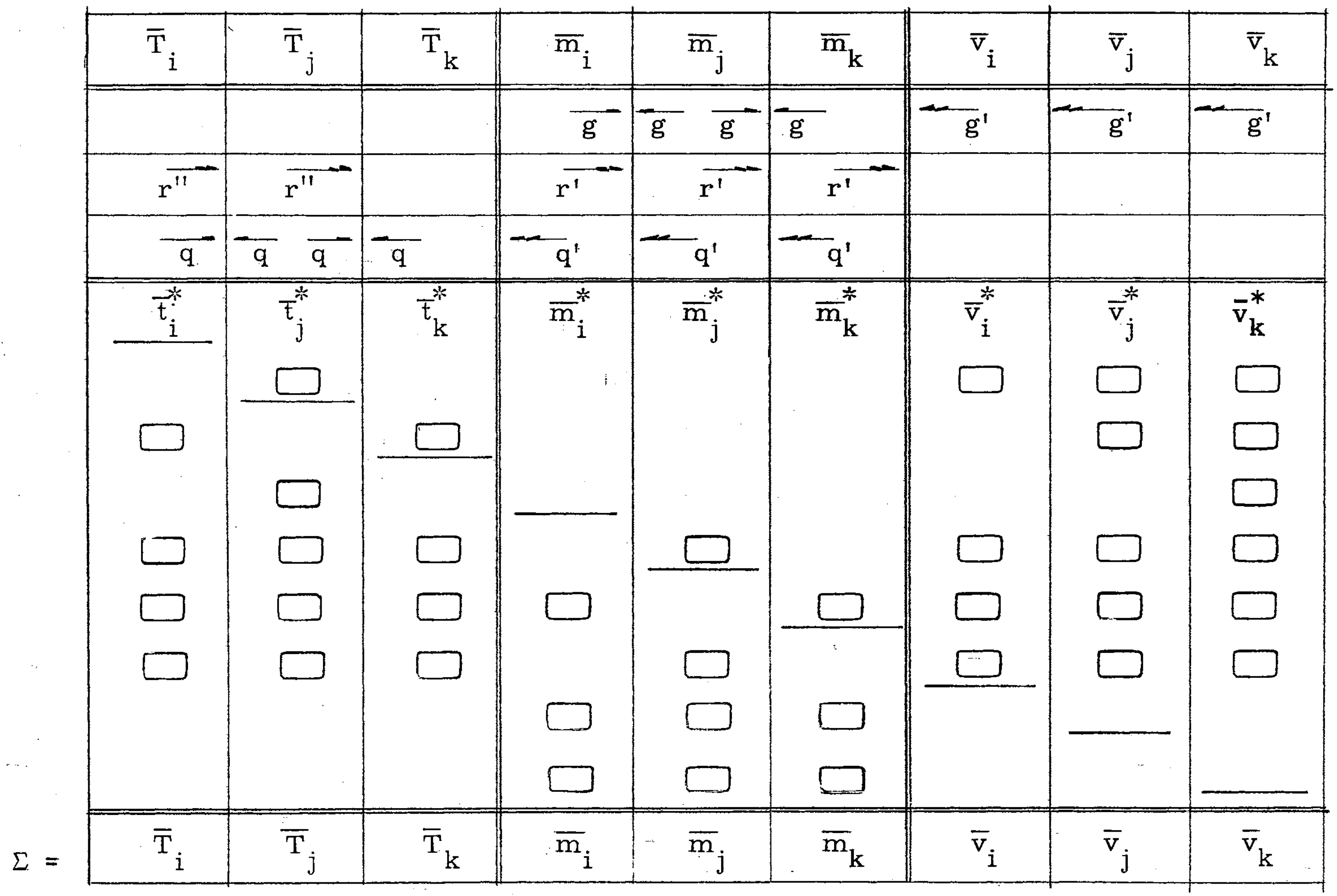


Fig. 5-1

Typical One Table Carry-Over

obtained. The starting values are $\overline{t}^*$, $\overline{v}^*$, and $\overline{m}^*$. The sum of each column is the solution for the corresponding redundant ($\overline{T}$, $\overline{v}$, or $\overline{m}$).

The sequence in which the carry-over procedure is accomplished is variable. However, all values must be carried over until the solution is obtained. In Fig. 5-1 the values are carried out of the columns proceeding from left to right and a line is drawn under these carried out values, thus accounting for the carry-overs.

5-2 Three Table Carry-Over

Equation (5-2) is resolved into three sub-matrix equations,

$$\begin{aligned} \left[q \right] \left[\overline{T} \right] &= \left[\overline{t}^* \right] && \text{(Shear Eq's.)} \\ \left[r \right] \left[\overline{v} \right] &= \left[\overline{v}^* \right] && \text{(Displacement Eq's.)} \qquad (5\text{-}3) \\ \left[g \right] \left[\overline{m} \right] &= \left[\overline{m}^* \right] && \text{(Moment Eq's.)} \end{aligned}$$

These equations can be solved by a general carry-over procedure within each sub-matrix and a carry-over procedure between each of the sub-matrices until Eq's. (5-3) are satisfied to a desired degree of accuracy.

The carry-over procedure within each of the sub-matrix equations can be conveniently presented in a tabular form (Fig. 5-2).

For an interior plate, the displacement is influenced only by the moments and shears; thus, a displacement sub-matrix carry-over is not required.

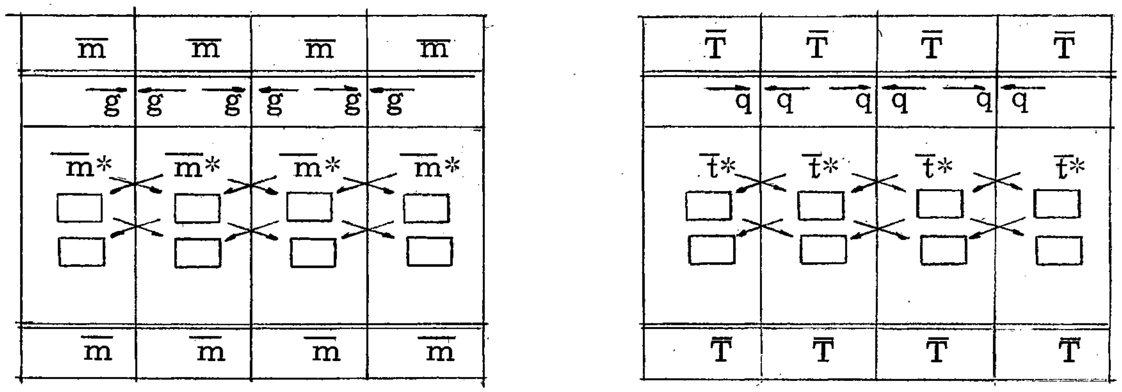


Fig. 5-2

Sub-Matrix Carry-Over

For a folded plate structure with edge members which are assumed to resist twisting, the displacements of the edge member and the adjacent plate influence each other. The sub-matrix displacement carry-over for this case is represented in tabular form (Fig. 5-3).

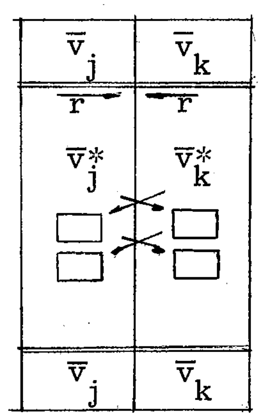


Fig. 5-3

Displacement Sub-Matrix Carry-Over

The solution of the deformation equations also requires compatibility between the sub-matrices. The carry-over procedure is represented in Fig. 5-4 with the carry-over factors shown. The starting values are the sums of each column in the sub-matrix carry-over (Fig's. 5-2, 3).

$$\begin{tikzcd}[column sep=huge] {\left[\,\bar{T}\,\right]} \arrow[rr, bend right, "r''"'] & {\left[\,\overline{m}\,\right]} \arrow[l, bend right, "q'"'] \arrow[r, bend right, "r'"'] & {\left[\,\bar{v}\,\right]} \arrow[l, bend right, "g''"'] \end{tikzcd}$$

Fig. 5-4

Carry-Over Between Sub-Matrices

This general procedure is repeated until the solution is obtained.

5-3 Two Table Carry-Over

When the displacement functions are substituted into the "Three Moment Equation with Displacement and Shear Influence", the Seven Moment Equation with Shear Influence" is obtained. The seven moment equations and the three shear flow equations comprise the necessary equations to solve the folded plate structure. The equations written in two sub-matrices are

$$\begin{aligned} \left[\,F\,\right]\left[\,\overline{m}\,\right] &= -\left[\,\Sigma\bar{T}^{*}\,\right] \quad \text{(Moment Eq's.)} \\ \left[\,Q\,\right]\left[\,\bar{T}\,\right] &= +\left[\,\Sigma\bar{T}^{*}\,\right] \quad \text{(Shear Eq's.)} \quad . \end{aligned} \qquad (5\text{-}4)$$

Rewriting Eq. (5-4) in a form more suitable for a carry-over procedure,

$$\begin{aligned} \left[g \right] \left[\overline{m} \right] &= \left[\overline{m}^* \right] \quad \text{(Moment Eq's.)} \\ \left[q \right] \left[\overline{T} \right] &= \left[\overline{t}^* \right] \quad \text{(Shear Eq's.)} \; . \end{aligned} \tag{5-5}$$

The general solution for the two sub-matrix formulation (Eq's. 5-5) can be accomplished in the same general manner as the three sub-matrix solution.

CHAPTER VI

EDGE PLATE AND EDGE BEAM EQUATIONS

6-1 General

The twisting moments m_{Tj} and the longitudinal moments are normally small in the thin interior plates and are commonly neglected. If a thickened edge plate (Fig. 6-1a) or edge beam (Fig. 6-5a) is utilized, the twisting moments and longitudinal moments become more appreciable and should be considered. The twisting moments and longitudinal moments in the edge member resist an additional horizontal edge force in a plane perpendicular to the plane of the plate (z direction), while the thin interior plates are assumed to resist only coplanar forces.

Two methods are presented: the plate theory for thickened edge plates, and the beam theory for edge members with a beam depth to thickness ratio. The equations are restricted to vertical edge members.

The plate equations (Eq's. 6-1 through 6-5) are derived from Levy's Solution for rectangular plates simply supported at two opposite edges (1, 2). The derivations are not shown since this thesis is concerned only with their application to the folded plate structure.

6-2 Edge Plate Equations

A vertical edge plate ij of thickness d_j and width $b_j = h_j$ is considered (Fig. 6-1a).

The plate equation for the horizontal edge force H_j at j due to a transverse moment m_j applied at j is (Fig. 6-1b)

$$H_j = m_j \zeta_j \tag{6-1}$$

where

$$\zeta_j = \frac{\alpha}{2} \frac{(3 \sinh^2 \alpha b_j - \alpha^2 b_j^2)}{(3 \sinh \alpha b_j \cosh \alpha b_j + \alpha b_j)} .$$

The plate equation for the slope $\left(\frac{\partial w_j}{\partial y}\right)^{m_j}_{y=o}$ at j due to a transverse moment m_j applied at j is (Fig. 6-1c)

$$\left(\frac{\partial w_j}{\partial y}\right)^{m_j}_{y=o} = m_j X_j \tag{6-2}$$

where

$$X_j = \frac{1}{2K_j \alpha} \frac{(3 \cosh^2 \alpha b_j + \alpha^2 b_j^2 + 1)}{(3 \sinh \alpha b_j \cosh \alpha b_j + \alpha b_j)}$$

$$K_j = \frac{Ed_j^3}{12} \quad \text{(flexural rigidity of plate neglecting Poisson's ratio)} .$$

From the geometry of the structure, the horizontal displacement w_{ji} of the edge plate written in terms of the transverse displacements is (Fig. 4-2e)

$$w_{ji} = \left(v_j \cot \gamma_j - \frac{v_k}{\sin \gamma_j}\right) . \tag{6-3}$$

The plate equation for the horizontal edge force H_j' at j due to a horizontal displacement w_{ji} is (Fig. 6-1d)

$$H_j' = w_{ji}\zeta_j' = (v_j \cot\gamma_j - \frac{v_k}{\sin\gamma_j})\,\zeta_j' \tag{6-4}$$

where

$$\zeta_j' = \frac{K_j\alpha^3}{2}\,\frac{(9\sinh^2\alpha b_j - \alpha^2 b_j^2)}{(3\sinh\alpha b_j\cosh\alpha b_j + \alpha b_j)} .$$

The plate equation for the slope $(\frac{\partial w_j}{\partial y})^{H_j}_{y=o}$ at j due to a horizontal edge force H_j applied at j is (Fig. 6-1e)

$$(\frac{\partial w_j}{\partial y})^{H_j}_{y=o} = -w_{ji}\zeta_j = -(v_j\cot\gamma_j - \frac{v_k}{\sin\gamma_j})\,\zeta_j . \tag{6-5}$$

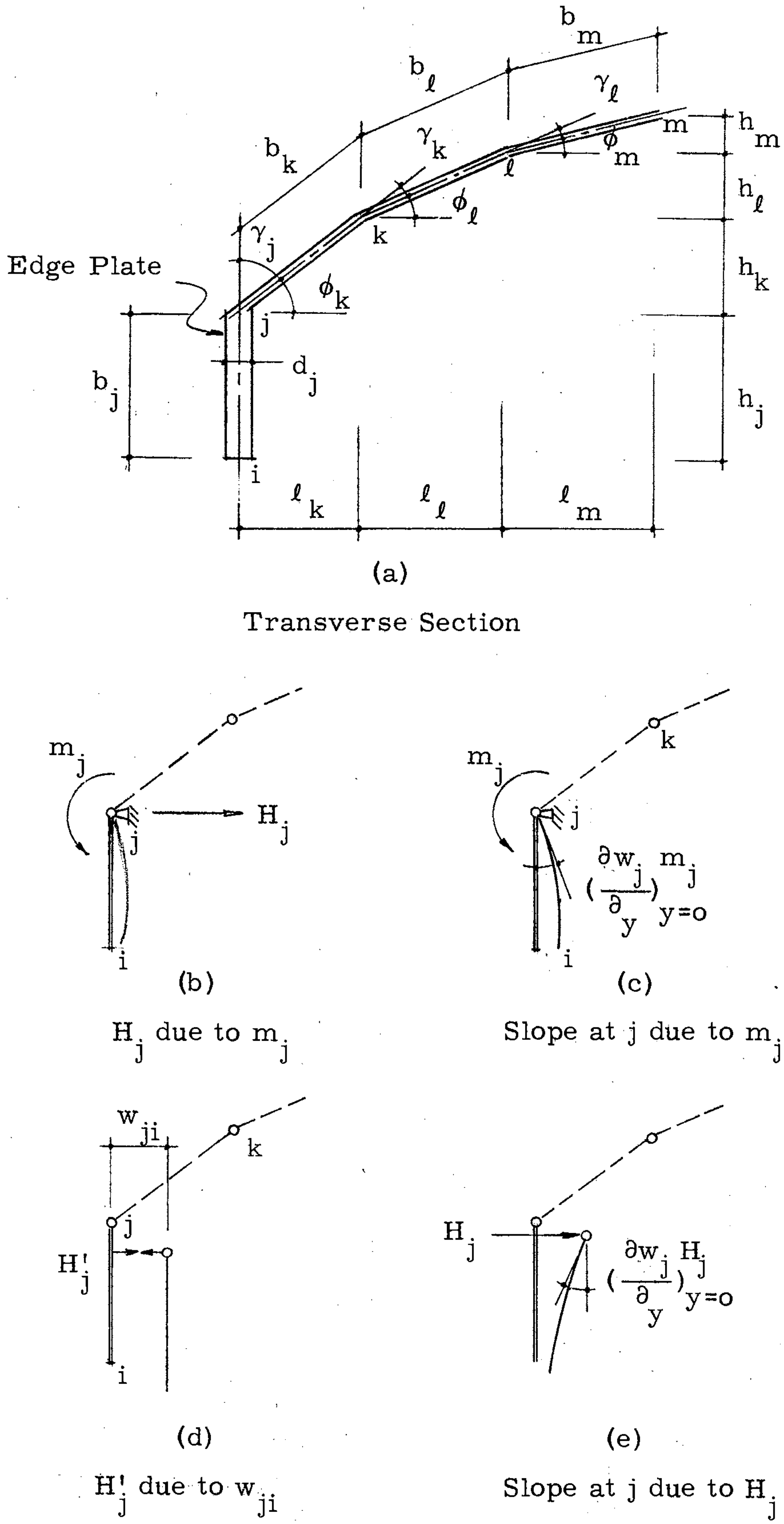
The slopes, displacements, and edge forces for the edge plate are defined from Eq's. (6-1, 2, 4, 5). The particular folded plate equations are obtained similarly as was presented in Chapters III and IV.

From the force polygon (Fig. 6-2), the transverse plate loads resulting from a horizontal edge force are obtained.

The plate loads due to the external ridge loads and the variation in the ridge loads caused by the transverse moments are

$$S_{ji}' = P_j\frac{\cos\phi_k}{\sin\gamma_j} = P_j$$

$$\Delta S_{ji} = \frac{m_k - m_j}{\ell_k} - (H_j + H_j')\tan\phi_k$$



(a)

Transverse Section

(b)

H_j due to m_j

(c)

Slope at j due to m_j

(d)

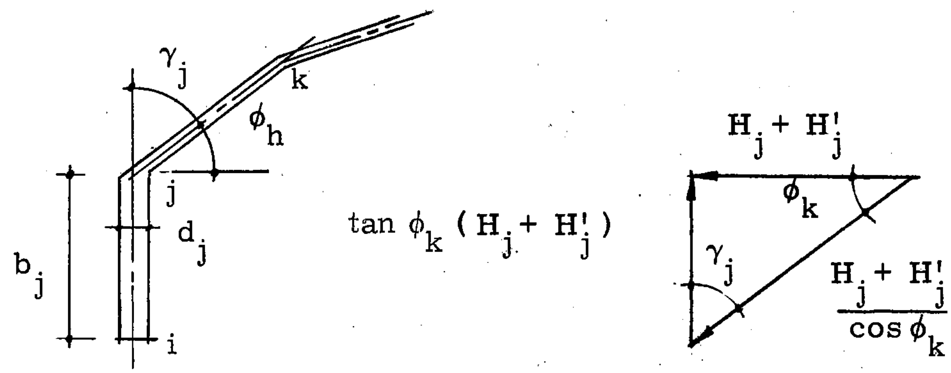
H'_j due to w_{ji}

(e)

Slope at j due to H_j

Fig. 6-1

Edge Plate Deformations



(a) Section

(b) Force Polygon

Fig. 6-2

Force Polygon for Edge Plate

$$S'_{jk} = P_j \frac{\cos\phi_j}{\sin\gamma_j} = 0 \tag{6-6}$$

$$\Delta S_{jk} = -\frac{H_j + H'_j}{\cos\phi_k}$$

$$S'_{kj} = P_k \frac{\cos\phi_\ell}{\sin\gamma_k}$$

$$\Delta S_{kj} = \left(\frac{m_j - m_k}{\ell_k} + \frac{m_\ell - m_k}{\ell_\ell}\right)\frac{\cos\phi_\ell}{\sin\gamma_k} .$$

The total plate loads are

$$S_j = S'_{ji} + \Delta S_{ji}$$

$$S_k = (S'_{kj} + \Delta S_{kj}) - (S'_{jk} + \Delta S_{jk}) . \tag{6-7}$$

The various functions represented by Fourier Series are

$$m_j = \Sigma_n \overline{m}_{jn} \sin\alpha x \qquad S'_j = \Sigma_n \overline{S}'_{jn} \sin\alpha x$$

$$H_j = \Sigma_n \overline{H}_{jn} \sin\alpha x \qquad \Delta S_j = \Sigma_n \Delta\overline{S}_{jn} \sin\alpha x \tag{6-8}$$

$$H'_j = \Sigma_n \overline{H}'_{jn} \sin\alpha x \qquad T_j = \Sigma_n \overline{T}_{jn} \cos\alpha x \ .$$

Since the principle of superposition holds, a typical term can be considered.

The equation for the redundant longitudinal shearing force at joint j is obtained by equating the longitudinal strain of adjoining plates (Fig. 3-2). The general three shear flow equation written for joint j is

$$\frac{1}{A_j} \overline{T}_i + 2\left(\frac{1}{A_j} + \frac{1}{A_k}\right) \overline{T}_j + \frac{1}{A_k} \overline{T}_k = \frac{3}{\alpha}\left(\frac{\overline{S}_j}{A_j b_j} + \frac{\overline{S}_k}{A_k b_k}\right) \tag{6-9}$$

but

$$\overline{T}_i = 0 \ .$$

Substituting the values of $\overline{S}_j$ and $\overline{S}_k$ from Eq's. (6-6, 7) and the rectangular plate solutions for H_j and H'_j from Eq's. (6-1, 4) yields the "Three Shear Flow Equation with Moment and Displacement Influence". At joint j, this equation is

$$\Sigma Q_j \overline{T}_j + Q_{kj} \overline{T}_k = Q'_{jj} \overline{m}_j + Q'_{kj} \overline{m}_k + Q'_{\ell j} \overline{m}_\ell + Q''_{jj} \overline{v}_j$$

$$+ Q''_{kj} \overline{v}_k + \Sigma \overline{T}^*_j \ . \tag{6-10}$$

Writing in carry-over form, Eq. (6-10) becomes

$$\overline{T}_j = + q_{kj}\overline{T}_k + q'_{jj}\overline{m}_j + q'_{kj}\overline{m}_k + q'_{\ell j}\overline{m}_\ell + q''_{jj}\overline{v}_j$$
$$+ q''_{kj}\overline{v}_k + \overline{t}_j^{\,*} . \qquad (6\text{-}11)$$

The values of the coefficients in Eq's. (6-10, 11) are tabulated in the Appendix (Table 5).

The equation for the redundant longitudinal shearing force at joint k is obtained by equating the longitudinal strains of adjoining plates as was accomplished at joint j . The general three shear flow equation written for joint k is

$$\frac{1}{A_k}\overline{T}_j + 2\left(\frac{1}{A_k} + \frac{1}{A_\ell}\right)\overline{T}_k + \frac{1}{A_\ell}\overline{T}_\ell = \frac{3}{\alpha}\left(\frac{\overline{S}_k}{A_k b_k} + \frac{\overline{S}_\ell}{A_\ell b_\ell}\right) . \qquad (6\text{-}12)$$

Substituting the values of $\overline{S}_k$ and $\overline{S}_\ell$ from Eq's. (6-6, 7), and the rectangular plate solutions for H_j and H'_j from Eq's. (6-1, 4) yields the "Three Shear Flow Equation with Moment and Displacement Influence". At joint k, this equation is

$$Q_{jk}\overline{T}_j + \Sigma Q_k\overline{T}_k + Q_{\ell k}\overline{T}_\ell = + Q'_{jk}\overline{m}_j + Q'_{kk}\overline{m}_k + Q'_{\ell k}\overline{m}_\ell$$
$$+ Q'_{mk}\overline{m}_m + Q''_{jk}\overline{v}_j + Q''_{kk}\overline{v}_k + \Sigma\,\overline{T}_k^{\,*}. \qquad (6\text{-}13)$$

Writing in carry-over form, Eq. (6-13) becomes

$$\overline{T}_k = + q_{jk}\overline{T}_j + q_{\ell k}\overline{T}_\ell + q'_{jk}\overline{m}_j + q'_{kk}\overline{m}_k + q'_{\ell k}\overline{m}_\ell + q'_{mk}\overline{m}_m$$
$$+ q''_{jk}\overline{v}_j + q''_{kk}\overline{v}_k + \overline{t}_k^{\,*} . \qquad (6\text{-}14)$$

The values of the coefficients in Eq's. (6-13) and (6-14) are tabulated in the Appendix (Table 5).

The equation for the redundant moment at joint j is obtained by summing the angular deformations at joint j equal to zero (Fig. 6-3);

$$0 = \bar{\tau}_{jk} + \bar{\psi}_{jk} + \bar{\theta}_{k} + \left(\frac{\partial \bar{w}_j}{\partial y}\right)_{y=o} \qquad (6\text{-}15)$$

where

$$\left(\frac{\partial \bar{w}_j}{\partial y}\right)_{y=o} = \left(\frac{\partial \bar{w}_j}{\partial y}\right)^{H_j}_{y=o} + \left(\frac{\partial \bar{w}_j}{\partial y}\right)^{m_j}_{y=o} \quad .$$

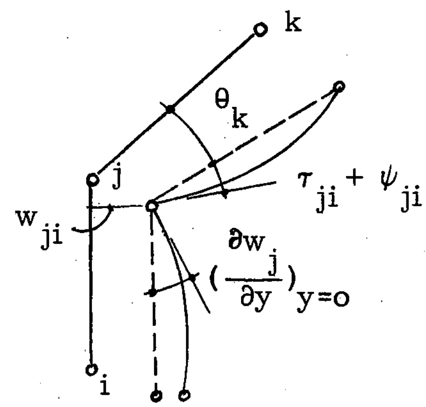


Fig. 6-3

Angular Deformations at Joint j

Substituting the values for $\bar{\tau}_{jk}$, $\bar{\psi}_{jk}$, $\bar{\theta}_{k}$ from Eq's. (4-7, 8, 9) and substituting the rectangular plate solutions for $\left(\frac{\partial \bar{w}_j}{\partial y}\right)_{y=o}$, Eq. (6-15) becomes

$$0 = \bar{\tau}_{jk} + G_{kj}\bar{m}_k + F_{jk}\bar{m}_j + X_{ji}\bar{m}_j$$

$$+ \frac{1}{b_k}\left[\bar{v}_k(\cot\gamma_j + \cot\gamma_k) - \frac{\bar{v}_j}{\sin\gamma_j} - \frac{\bar{v}_\ell}{\sin\gamma_k}\right] - \left(\bar{v}_j\cot\gamma_j - \frac{\bar{v}_k}{\sin\gamma_j}\right)\zeta_j \ .$$

Rewriting, the resulting equation is the "Three Moment Equation with Displacement Influence" at joint j .

$$0 = \Sigma F_j \overline{m}_j + G_{kj} \overline{m}_k + G'_{jj} \overline{v}_j + G'_{kj} \overline{v}_k + G'_{\ell j} \overline{v}_\ell + \Sigma \overline{\tau}_j^* . \qquad (6\text{-}16)$$

Writing in carry-over form, Eq. (6-16) becomes

$$\overline{m}_j = + g_{kj} \overline{m}_k + g'_{jj} \overline{v}_j + g'_{kj} \overline{v}_k + g'_{\ell j} \overline{v}_\ell + \overline{m}_j^* . \qquad (6\text{-}17)$$

The values of the coefficients in Eq's. (6-16, 17) are tabulated in the Appendix (Table 6).

The equation for the redundant displacement $\overline{v}_j$ of plate ij is obtained by substituting into the general displacement equation (Eq. 6-18) the values of $\overline{S}_j$ and $\Delta\overline{S}_j$ from Eq's. (6-6, 7) ,

$$EI_j \alpha^4 \overline{v}_j = \overline{S}_j + \Delta \overline{S}_j - (\overline{T}_i + \overline{T}_j) \frac{\alpha b_j}{2} . \qquad (6\text{-}18)$$

Substituting, the "Displacement Equation with Moment and Shear Influence" for plate ij is

$$\Sigma R_j \overline{v}_j = + R_{kj} \overline{v}_k + R'_{jj} \overline{m}_j + R'_{kj} \overline{m}_k + R''_{jj} \overline{T}_j + \Sigma \overline{V}_j^* . \qquad (6\text{-}19)$$

Writing in carry-over form, Eq. (6-19) becomes

$$\overline{v}_j = + r_{kj} \overline{v}_k + r'_{jj} \overline{m}_j + r'_{kj} \overline{m}_k + r''_{jj} \overline{T}_j + \overline{v}_j^* . \qquad (6\text{-}20)$$

The values of the coefficients in Eq's. (6-19, 20) are tabulated in the Appendix (Table 7).

The equation for the redundant displacement $\overline{v}_k$ of plate jk is obtained by substituting into the general displacement equation (Eq. 6-21).

the values of $\overline{S}_k$ and $\Delta \overline{S}_k$ from Eq's. (6-6, 7),

$$EI_k \alpha^4 \overline{v}_k = \overline{S}_k + \Delta \overline{S}_k - (\overline{T}_j + \overline{T}_k) \frac{\alpha b_k}{2} \quad . \tag{6-21}$$

Substituting, the "Displacement Equation with Moment and Shear Influence" for plate jk is

$$\Sigma R_k \overline{v}_k = + R_{jk} \overline{v}_j + R'_{jk} \overline{m}_j + R'_{kk} \overline{m}_k + R'_{\ell k} \overline{m}_\ell + R''_{jk} \overline{T}_j + R''_{kk} \overline{T}_k + \Sigma \overline{V}_k^* \quad . \tag{6-22}$$

Writing in carry-over form, Eq. (6-22) becomes

$$\overline{v}_k = + r_{jk} \overline{v}_j + r'_{jk} \overline{m}_j + r'_{kk} \overline{m}_k + r'_{\ell k} \overline{m}_\ell + r''_{jk} \overline{T}_j + r''_{kk} \overline{T}_k + \overline{v}_k^* \quad , \tag{6-23}$$

The values of the coefficients in Eq's. (6-22, 23) are tabulated in the Appendix (Table 7).

6-3 Edge Beam Equations

Consider a vertical edge beam ij of thickness d_j and width $b_j = h_j$ (Fig. 6-5a). The member ij is considered to act as a beam and is capable of resisting torsion about the x axis and bending about the y and z axes. The same procedure is followed as was presented for the plate theory.

The equation for the angle of twist $d\theta$ for a prismatic bar of length dx is

$$d\theta_j = \frac{m_{tx}\,dx}{J_j G}$$

or

$$m_{tx} = \frac{d\theta_j}{dx} J_j G \tag{6-24}$$

where J is the torsional constant and G is the modulus of rigidity of the material. Referring to Fig. 6-4, the equation of moment equilibrium for the differential element is

$$m_{tx} + \frac{\partial m_{tx}}{\partial x} dx - m_{tx} - m_j dx = 0$$

or

$$m_j = \frac{\partial m_{tx}}{\partial x} . \tag{6-25}$$

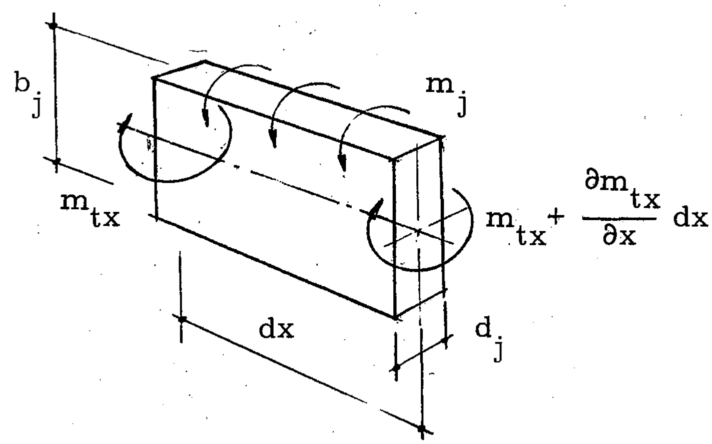


Fig. 6-4

Differential Element in Torsion

Differentiating Eq. (6-24) with respect to x and substituting into Eq. (6-25)

$$m_j = J_j G \frac{\partial^2 \theta_j}{\partial x^2} \quad . \tag{6-26}$$

Writing Eq. (6-26) for a typical term of its Fourier Series, the equation becomes

$$\overline{m}_j \sin \alpha x = - J_x G \frac{\partial^2 \overline{\theta}_j}{\partial x^2} \sin \alpha x \quad .$$

Integrating twice with respect to x, the slope θ_j due to a moment applied at j is (Fig. 6-5b)

$$\theta_j = m_j \frac{1}{\alpha^2 J_j G} \quad . \tag{6-27}$$

The horizontal edge force H_j at j due to a moment m_j applied at j is (Fig. 6-5c)

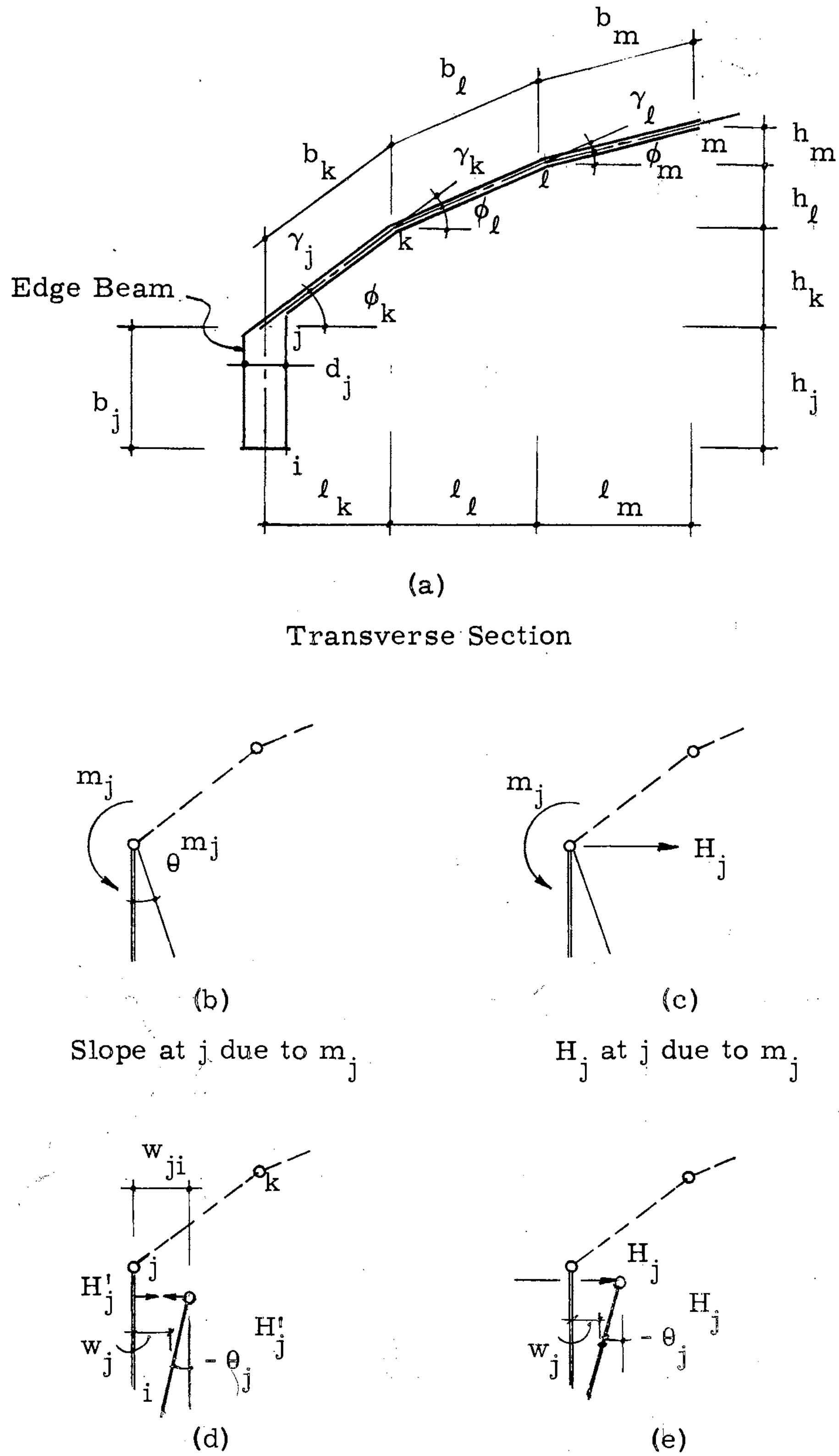
$$H_j = m_j \frac{2}{b_j} \quad . \tag{6-28}$$

From the geometry of the structure, the horizontal displacement w_{ji} of the edge beam can be written in terms of the transverse displacements (Fig. 4-2e).

$$w_{ji} = (v_j \cot \gamma_j - \frac{v_k}{\sin \gamma_j}) \quad . \tag{6-29}$$

The equation for the horizontal edge force H'_j due to a horizontal displacement w_{ji} is obtained from the differential equation

$$\frac{d^2 w_{ji}}{dx^2} = - \frac{M_y}{EI_{jy}} \tag{6-30}$$



(a)

Transverse Section

(b)

Slope at j due to m_j

(c)

H_j at j due to m_j

(d)

H'_j due to w_{ji}

(e)

Slope at j due to H_j

Fig. 6-5

Edge Beam Deformations

where M_y is the bending moment about the y axis. M_y, Q_y, and w_{ji} expressed by Fourier Series are

$$w_{ji} = \Sigma_n \bar{w}_{jin} \sin\alpha x \qquad M_y = \Sigma_n \bar{M}_{yn} \sin\alpha x \qquad Q_y = \Sigma_n \bar{Q}_{yn} \cos\alpha x \ .$$

Also

$$dM_y = Q_y dx \quad , \qquad dQ_y = -H'_j dx \tag{6-31}$$

where Q_y is the horizontal shear. Substituting the values of M_y and Q_y into Eq's. (6-31), and integrating with respect to x,

$$M_y = \frac{1}{\alpha} Q_y \quad , \qquad Q_y = -\frac{1}{\alpha} H'_j \ . \tag{6-32}$$

Differentiating twice with respect to x, Eq. (6-31) becomes

$$\frac{d^2 w_{ji}}{dx^2} = -\alpha^2 \bar{w}_{ji} \sin\alpha x \ . \tag{6-33}$$

From Eq's. (6-31, 32, 33), the horizontal edge force due to a horizontal displacement is (Fig. 6-5d)

$$H'_j = w_j \, EI_{jy} \, \alpha^4 = (w_{ji} + \theta_j^{H'_j} \frac{b_j}{2}) \, EI_{jy} \alpha^4 \ .$$

Substituting w_{ji} from Eq. (6-27, 29) and $\theta_j^{H'_j}$ from (6-27, 28)

$$H'_j = (v_j \cot\gamma_j - \frac{v_k}{\sin\gamma_j}) \frac{(EI_{jy}\alpha^4)^2}{4JG\alpha^2 + b_j^2 \, EI_{jy} \alpha^4} \ . \tag{6-34}$$

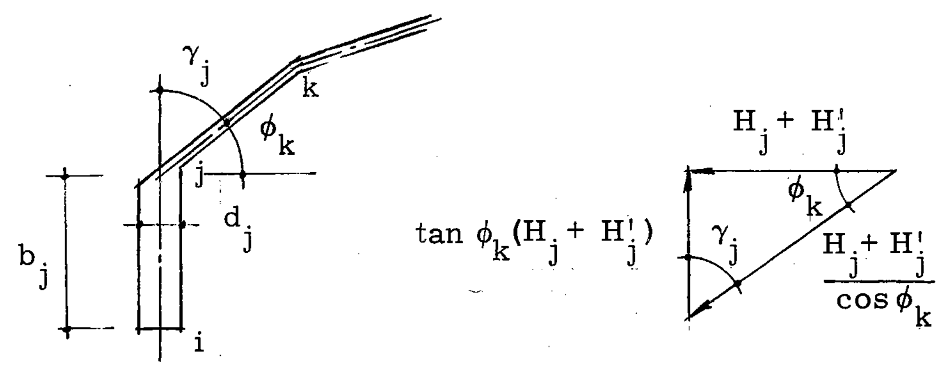
The beam equation for the slope at j $\theta_j^{H_j}$ of the beam ij due to a horizontal edge force H_j applied at j from Eq. (6-27) (Fig. 6-5e) is

$$\theta_j^{H_j} = -\frac{H_j \frac{b_j}{2}}{JG\alpha^2} = -\frac{(w_{ji} + \theta_j^{H_j} \frac{b_j}{2}) \frac{b_j}{2} \, EI_{jy} \alpha^4}{JG\alpha^2} \ .$$

Substituting w_{ji} from Eq. (6-29), and θ_j from (6-27)

$$\theta_j^{H_j} = -\left(v_j \cot\gamma_j - \frac{v_k}{\sin\gamma_j}\right) \frac{EI_{jy}\,\alpha^4}{(JG\alpha^2 + \frac{1}{2} b_j^2\, EI_{jy}\,\alpha^4)} \qquad (6\text{-}35)$$

With the slopes, displacements, and edge forces defined, the folded plate equations are obtained similarly to that presented for the edge plate. The transverse plate loads due to the horizontal edge force are obtained from the force polygon at joint j (Fig. 6-6).



(a)

Section

(b)

Force Polygon

Fig. 6-6

Force Polygon for Edge Beam

The various functions are represented by the Fourier Series similarly as for the edge plate (Eq's. 6-8) and also

$$\theta_j = \Sigma_n \overline{\theta}_{jn} \sin\alpha x . \qquad (6\text{-}36)$$

The plate loads are similar to Eq's. (6-6, 7) except the beam functions are applicable in place of the plate functions.

The general three shear flow equation written for joint j is

$$\frac{1}{A_j}\overline{T}_i + 2\left(\frac{1}{A_j} + \frac{1}{A_k}\right)\overline{T}_j + \frac{1}{A_k}\overline{T}_k = \frac{3}{\alpha}\left(\frac{\overline{S}_j}{A_j b_j} + \frac{\overline{S}_k}{A_k b_k}\right) \tag{6-37}$$

but

$$\overline{T}_i = 0.$$

Substituting the values of S_j and S_k from Eq's. (6-6, 7) and the values of H_j and H'_j from Eq's. (6-28, 34), the "Shear Flow Equation with Moment and Displacement Influence" at joint j is

$$\Sigma Q_j \overline{T}_j + Q_{kj}\overline{T}_k = Q'_{jj}\overline{m}_j + Q'_{kj}\overline{m}_k + Q'_{\ell j}\overline{m}_\ell + Q''_{jj}\overline{v}_j + Q''_{kj}\overline{v}_k + \Sigma\overline{T}^*_j . \tag{6-38}$$

Writing in carry-over form, Eq. (6-38) becomes

$$\overline{T}_j = + q_{kj}\overline{T}_k + q'_{jj}\overline{m}_j + q'_{kj}\overline{m}_k + q'_{\ell j}\overline{m}_\ell + q''_{jj}\overline{v}_j + q''_{kj}\overline{v}_k + \overline{t}^*_j . \tag{6-39}$$

The values of the coefficients of Eq's. (6-38) and (6-39) are tabulated in the Appendix (Table 8).

The general three shear flow equation written for joint k is

$$\frac{1}{A_k}\overline{T}_j + 2\left(\frac{1}{A_k} + \frac{1}{A_\ell}\right)\overline{T}_k + \frac{1}{A_\ell}\overline{T}_\ell = \frac{3}{\alpha}\left(\frac{\overline{S}_k}{A_k b_k} + \frac{\overline{S}_\ell}{A_\ell b_\ell}\right) . \tag{6-40}$$

Substituting similarly as for joint j, the "Three Shear Flow Equation with Displacement Influence" at joint k is

$$Q_{jk}\overline{T}_j + \Sigma\, Q_k\overline{T}_k + Q_{\ell k}\overline{T}_\ell \;=\; + Q'_{jk}\overline{m}_j + Q'_{kk}\overline{m}_k + Q'_{\ell k}\overline{m}_\ell$$

$$+ Q'_{mk}\overline{m}_m + Q''_{jk}\overline{v}_j + Q''_{kk}\overline{v}_k + \Sigma\,\overline{T}^*_k \qquad (6\text{-}41)$$

Writing in carry-over form, Eq. (6-41) becomes

$$\overline{T}_k = + q_{jk}\overline{T}_j + q_{\ell k}\overline{T}_\ell + q'_{jk}\overline{m}_j + q'_{kk}\overline{m}_k + q'_{\ell k}\overline{m}_\ell + q'_{mk}\overline{m}_m$$

$$+ q''_{jk}\overline{v}_j + q''_{kk}\overline{v}_k + \overline{t}^*_k \;. \qquad (6\text{-}42)$$

The values of the coefficients in Eq's. (6-41, 42) are tabulated in the Appendix (Table 7).

Angular deformations are summed to zero for joint j (Fig. 6-7).

$$0 = \overline{\tau}_{jk} + \overline{\psi}_{jk} + \overline{\theta}_k + \overline{\theta}_j \;. \qquad (6\text{-}43)$$

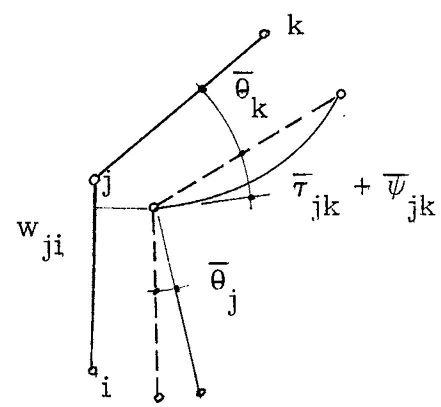


Fig. 6-7

Angular Deformations at Joint j

Substituting the values for $\overline{\tau}_{jk}$, $\overline{\psi}_{jk}$, and $\overline{\theta}_k$ from Eq's. (4-7, 8, 9) and the values of $\overline{\theta}_j$ from Eq's. (6-27, 35), Eq. (6-43) becomes

$$0 = \bar{\tau}_{jk} + G_{kj}\overline{m}_k + F_{jk}\overline{m}_j + \frac{1}{\alpha^2 J_j G}\,\overline{m}_j$$

$$- \frac{1}{b_k}\left[\bar{v}_k(\cot\gamma_j + \cot\gamma_k) - \frac{\bar{v}_j}{\sin\gamma_j} - \frac{\bar{v}_\ell}{\sin\gamma_k}\right]$$

$$- (\bar{v}_j \cot\gamma_j - \frac{\bar{v}_k}{\sin\gamma_j})\,\frac{2}{b_j\alpha^2 J_j G}\,.$$

Rewriting, the "Three Moment Equation with Displacement Influence" at joint j is

$$0 = \Sigma F_j\overline{m}_j + G_{kj}\overline{m}_k + G'_{jj}\bar{v}_j + G'_{kj}\bar{v}_k + G'_{\ell j}\bar{v}_\ell + \Sigma\bar{\tau}^*_j\,. \tag{6-44}$$

Writing in carry-over form, Eq. (6-44) becomes

$$\overline{m}_j = + g_{kj}\overline{m}_k + g'_{jj}\bar{v}_j + g'_{kj}\bar{v}_k + g'_{\ell j}\bar{v}_\ell + \overline{m}^*_j. \tag{6-45}$$

The values of the coefficients in Eq's. (6-44, 45) are tabulated in the Appendix (Table 9).

The general displacement equation for member ij is

$$EI_j\alpha^4\bar{v}_j = \bar{S}_j + \Delta\bar{S}_j - (\bar{T}_i + \bar{T}_j)\,\frac{\alpha b_j}{2}\,. \tag{6-46}$$

Substituting the values of $\bar{S}_j$ and $\Delta\bar{S}_j$ from Eq's. (6-6, 7) and the beam equation values for H_j and H'_j from Eq's. (6-28, 34), the "Displacement Equation with Moment and Shear Influence" for the edge beam ij is

$$\Sigma R_j\bar{v}_j = + R_{kj}\bar{v}_k + R'_{jj}\overline{m}_j + R'_{kj}\overline{m}_k + R''_{jj}\bar{T}_j + \Sigma\bar{V}^*_j\,. \tag{6-47}$$

Writing in carry-over form, Eq. (6-47) becomes

$$\overline{v}_j = + r_{kj}\overline{v}_k + r'_{jj}\overline{m}_j + r'_{kj}\overline{m}_k + r''_{jj}\overline{T}_j + \overline{v}_j^*. \tag{6-48}$$

The values of the coefficients in Eq's. (6-47, 48) are tabulated in the Appendix (Table 10).

The general displacement equation for plate jk is

$$EI_k\alpha^4\overline{v}_k = \overline{S}_k + \Delta\overline{S}_k - (\overline{T}_j + \overline{T}_k)\frac{\alpha b_k}{2} \quad . \tag{6-49}$$

Substituting $\overline{S}_k$ and $\Delta\overline{S}_k$ from Eq's. (6-6, 7), and also the edge beam values for H_j and H'_j from Eq's. (6-28, 34), the "Displacement Equation with Moment and Displacement Influence" for plate jk is

$$\Sigma R_k\overline{v}_k = + R_{jk}\overline{v}_j + R'_{jk}\overline{m}_j + R'_{kk}\overline{m}_k + R'_{\ell k}\overline{m}_\ell + R''_{jk}\overline{T}_j + R''_{kk}\overline{T}_k + \Sigma\overline{V}_k^*. \tag{6-50}$$

Writing in carry-over form, Eq. (6-50) becomes

$$\overline{v}_k = + r_{jk}\overline{v}_j + r'_{jk}\overline{m}_j + r'_{kk}\overline{m}_k + r'_{\ell k}\overline{m}_\ell + r''_{jk}\overline{T}_j + r''_{kk}\overline{T}_k + \overline{v}_j^* . \tag{6-51}$$

The values of the coefficients in Eq's. (6-50, 51) are tabulated in the Appendix (Table 10).

CHAPTER VII

APPLICATION

7-1 General

The simply supported folded plate structure with thickened vertical edge plates shown in Fig. 7-1 is analyzed for dead load. The metric system of measures is utilized for comparison of results with those of Girkmann (10). Only the first term of the Fourier Series representing the various functions is shown. For a complete analysis several additional terms should be considered.

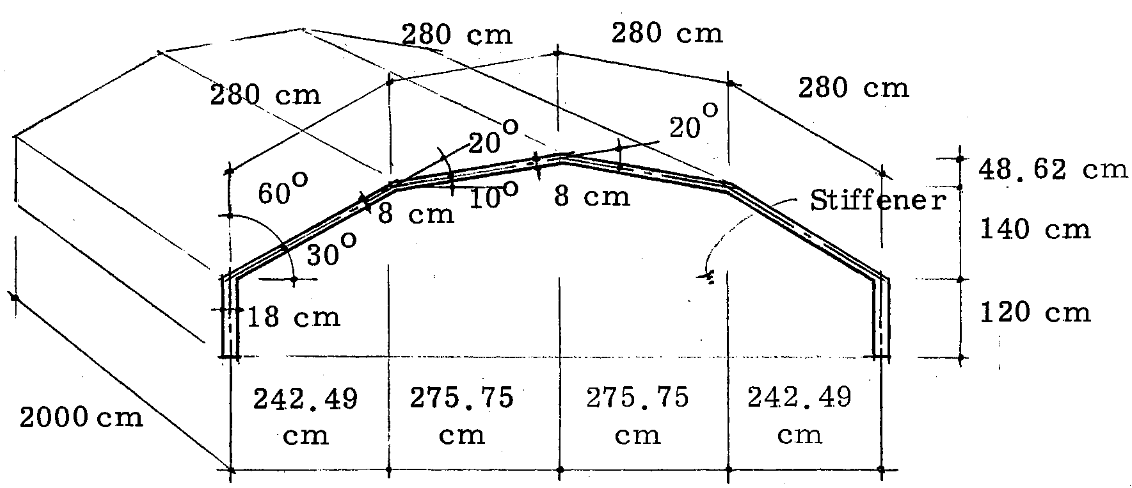


Fig. 7-1

Transverse Section

Geometry:

$d_1 = 18$ cm. $\quad b_1 = 120$ cm. $\quad A_1 = 2160$ cm.^{2}

$d_2 = d_3 = 8$ cm. $\quad b_2 = b_3 = 280$ cm. $\quad A_2 = A_3 = 2240$ cm.^{2}

$I_1 = 2.5920$ cm.^{4} $\quad E = .210$ kg/cm.^{2}

$I_2 = I_3 = 14.6347$ cm.^{4}

$\phi_1 = 90^o \quad \gamma_1 = 60^o$

$\phi_2 = 30^o \quad \gamma_2 = 20^o$

$\phi_3 = 10^o \quad \gamma_3 = 20^o$

$\gamma_4 = 20^o$

Loads:

plate 1	concrete	$.18 \times .24$	$= .0432$
	soffit		$= .0022$
		p_1	$= .0454$ kg/cm.^{2}
plate 2	concrete	$.08 \times .24$	$= .0192$
	soffit		$= .0022$
	$p_3 =$	p_2	$= .0214$ kg/cm.^{2}

$$P_1 \doteq .0454(120) + \frac{1}{2}(.0214)(280) + 22(\text{gutter}) = 8.7 \text{ kg/cm.}$$

$$P_2 = \frac{1}{2}(.0214)(280)2 = 6.0 \text{ kg/cm.}$$

Representing the loads by Fourier Series,

$$\overline{p}_2 = \overline{p}_3 = \frac{4}{\pi}(.0214) = .02724 \text{ kg/cm.}^2$$

$$\overline{P}_1 = \frac{4}{\pi}(8.7) = 11.0772 \text{ kg/cm.}$$

$$\overline{P}_2 = \overline{P}_3 = \frac{4}{\pi}(6.0) = 7.6394 \text{ kg/cm.}$$

Edge Plate Constants: (Eq's. 6-1, 2, 4)

$\alpha = \frac{n\pi}{L} = 1.5708 \times 10^{-3}$ 1/cm.

$\alpha b_1 = (1.5708)(120) = .1885$

$\zeta_1 = 7.4021 \times 10^{-5}$

$\zeta_1' = 7.4232 \times 10^{-2}$

$X_1 = 1.6840 \times 10^{-5}$

7-2 Influence Coefficients

The coefficients for the three sets of deformation equations are evaluated from the tables in the Appendix.

Shear Flow Influence Coefficients: (Table 5)

Joint 1

$\Sigma Q_1 = +1.81876$ $\quad Q_{21} = +.44642$ $\quad Q'_{11} = +5.7180$

$Q'_{21} = -37.5697 \times 10^{-6}$ $\quad Q'_{31} = +31.7973 \times 10^{-6}$ $\quad Q''_{11} = -31.6260 \times 10^{-6}$

$Q''_{21} = +63.2485 \times 10^{-6}$ $\quad \Sigma\bar{T}_1^* = +148.600 \times 10^{-3}$

Joint 2

$\Sigma Q_2 = +1.78568 \times 10^{-3}$ $\quad Q_{12} = +.44642 \times 10^{-3}$ $\quad Q'_{12} = +4.6215 \times 10^{-6}$

$Q'_{22} = +23.6009 \times 10^{-6}$ $\quad Q'_{32} = -59.7594 \times 10^{-6}$ $\quad Q'_{42} = +31.7973 \times 10^{-6}$

$Q''_{12} = +150.6950 \times 10^{-6}$ $\quad Q''_{22} = -301.3918 \times 10^{-6}$ $\quad \Sigma\bar{T}_2^* = +75.060 \times 10^{-3}$

Moment Influence Coefficients: (Tables 3, 7)

Joint 1

$\Sigma F_1 = +27.2567 \times 10^{-6}$ $G_{21} = +5.2083 \times 10^{-6}$ $G'_{11} = -4.1667 \times 10^{-3}$

$G'_{21} = +11.9600 \times 10^{-3}$ $G'_{31} = -10.4422 \times 10^{-6}$ $\Sigma \overline{\tau}^*_1 = +2409.11 \times 10^{-6}$

Joint 2

$\Sigma F_2 = +20.8333 \times 10^{-6}$ $G_{12} = +5.2083 \times 10^{-6}$ $G_{32} = +5.2083 \times 10^{-6}$

$G'_{12} = +4.1239 \times 10^{-3}$ $G'_{22} = -22.3167 \times 10^{-3}$ $G'_{32} = +30.0672 \times 10^{-3}$

$G'_{42} = -10.4422 \times 10^{-3}$ $\Sigma \overline{\tau}^*_2 = +5148.65 \times 10^{-6}$

Joint 3

$\Sigma F_3 = +20.8333 \times 10^{-6}$ $G_{23} = +5.2083 \times 10^{-6}$ $G_{43} = +5.2083 \times 10^{-6}$

$G'_{23} = +10.4422 \times 10^{-3}$ $G'_{33} = -30.0672 \times 10^{-3}$ $G'_{43} = +30.0672 \times 10^{-6}$

$G'_{53} = -10.4422 \times 10^{-3}$ $\Sigma \overline{\tau}^*_3 = +5479.08 \times 10^{-6}$

Displacement Influence Coefficients: (Tables 2, 6)

Plate 1

$\Sigma R_1 = +3.3386$ $R_{21} = +.0495$ $R'_{11} = -4.1979 \times 10^{-3}$

$R'_{21} = +4.1239 \times 10^{-3}$ $R''_{11} = -94.248 \times 10^{-3}$ $\Sigma \overline{V}^*_1 = +11.0772$

Plate 2

$\Sigma R_2 = +18.8095$ $R_{12} = +.0495$ $R'_{12} = +11.9598 \times 10^{-3}$

$R'_{22} = -22.3164 \times 10^{-3}$ $R'_{32} = +10.4422 \times 10^{-3}$ $R''_{12} = -219.9120 \times 10^{-3}$

$R''_{22} = -219.9120 \times 10^{-3}$ $\Sigma \overline{V}^*_2 = +21.9968$

Plate 3

$\Sigma R_3 = +18.7105$ $\quad R'_{13} = -10.4422 \times 10^{-3}$ $\quad R'_{23} = +30.0669 \times 10^{-3}$

$R'_{33} = -30.0669 \times 10^{-3}$ $\quad R'_{43} = +10.4422 \times 10^{-3}$ $\quad R''_{23} = -219.912 \times 10^{-3}$

$R''_{33} = -219.912 \times 10^{-3}$ $\quad \Sigma \overline{V}^*_3 = +2.6513$

7-3 Carry-Over Equations

From the conditions of symmetry it is known that

$\overline{T}_1 = -\overline{T}_5$ $\quad \overline{v}_1 = -\overline{v}_6$ $\quad \overline{m}_1 = \overline{m}_5$

$\overline{T}_2 = -\overline{T}_4$ $\quad \overline{v}_2 = -\overline{v}_5$ $\quad \overline{m}_2 = \overline{m}_4$

$\overline{T}_3 = 0$ $\quad \overline{v}_3 = -\overline{v}_4$

The equations written in carry-over form including the modifications for symmetry are

$$\overline{T}_1 = -.24545\,\overline{T}_2 + .003144\,\overline{m}_1 - .020657\,\overline{m}_2 + .017483\,\overline{m}_3 - .017389\,\overline{v}_1 + .034776\,\overline{v}_2 + 81.7040$$

$$\overline{T}_2 = -.25000\,\overline{T}_1 + .002588\,\overline{m}_1 + .031024\,\overline{m}_2 - .033466\,\overline{m}_3 + .084391\,\overline{v}_1 - .168783\,\overline{v}_2 + .042034$$

$$\overline{m}_1 = -.1911\,\overline{m}_2 + 152.86\,\overline{v}_1 - 438.79\,\overline{v}_2 + 383.10\,\overline{v}_3 - 88.386$$

$$\overline{m}_2 = -.2500\,\overline{m}_1 - .2500\,\overline{m}_3 - 197.95\,\overline{v}_1 + 1071.20\,\overline{v}_2 - 1944.46\,\overline{v}_3 - 247.14$$

$$\overline{m}_3 = -.5000\,\overline{m}_2 - 1002.46\,\overline{v}_2 + 2886.46\,\overline{v}_3 - 263.00$$

$$\bar{v}_1 = +.0148\,\bar{v}_2 - .001257\,\bar{m}_1 + .001235\,\bar{m}_2 - .028230\,\bar{T}_1 + 3.3179$$

$$\bar{v}_2 = +.0026\,\bar{v}_1 + .000636\,\bar{m}_1 - .001186\,\bar{m}_2 + .000555\,\bar{m}_3 - .001692\,\bar{T}_1 - .011692\,\bar{T}_2 + 1.1695$$

$$\bar{v}_3 = -.000558\,\bar{m}_1 + .002165\,\bar{m}_2 - .001607\,\bar{m}_3 - .011753\,\bar{T}_2 + .1418\,.$$

Because of the large displacement to moment carry-over values the convergency will be difficult to obtain; therefore, the displacement functions are eliminated. The $\bar{v}_1$ and $\bar{v}_2$ equations are solved simultaneously and the displacement equations are substituted into the shear and moment equations. Thus, the modified carry-over equations are

$$\bar{T}_1 = -.2459\,\bar{T}_2 + .00319\,\bar{m}_1 - .02072\,\bar{m}_2 + .01750\,\bar{m}_3 + 81.694$$

$$\bar{T}_2 = -.2509\,\bar{T}_1 + .00238\,\bar{m}_1 + .03139\,\bar{m}_2 - .03363\,\bar{m}_3 + 42.200$$

$$\bar{m}_1 = +.7988\,\bar{m}_2 - .5101\,\bar{m}_3 + .4882\,\bar{T}_1 + .3573\,\bar{T}_2 - 24.509$$

$$\bar{m}_2 = +.2619\,\bar{m}_1 + .5162\,\bar{m}_3 - 1.0391\,\bar{T}_1 + 1.5427\,\bar{T}_2 + 11.763$$

$$\bar{m}_3 = -.3624\,\bar{m}_1 + 1.1195\,\bar{m}_2 + 1.9039\,\bar{T}_1 - 3.5844\,\bar{T}_2 - 167.035\,.$$

7-4 Carry-Over Procedure

The solution of the two sets of deformation equations is accomplished by a one table carry-over procedure (Fig. 7-2).

7-5 Numerical Control

The accuracy of the final answers can be checked by substituting

	$\overline{T}_1$	$\overline{T}_2$	$\overline{m}_1$	$\overline{m}_2$	$\overline{m}_3$
q's		- .2459	+ .00319	- .02072	+ .01750
q's	- .2509		+ .00238	+ .03139	- .03363
g's	+ .4882	+ .3573		- .7988	- .5101
g's	- 1.0391	+ 1.5427	+ .2619		+ .5163
g's	+ 1.9039	- 3.5844	- .3624	+ 1.1195	
$\overline{t}$*, $\overline{m}$*	+81.694	+42.200	-24.509	+ 11.763	-167.035
		-20.495	+39.876	- 84.881	+155.521
	- 5.337		+ 7.755	+ 33.486	- 77.803
	+ .074	- .055		+ 6.056	- 8.379
	+ .696	- 1.054	-26.819		- 37.589
	- 2.368	+ 4.550	+69.005	- 69.830	
		+ 1.740	- 3.385	+ 7.310	- 13.203
	- 1.274		+ 1.851	+ 7.992	- 18.569
	+ .130	- .097		+ 10.648	- 14.731
	+ .909	- 1.377	-35.050		- 49.125
	- 1.674	+ 3.216	+48.777	- 49.361	
		+ .479	- .932	+ 1.983	- 3.633
	- .546		+ .793	+ 3.426	- 7.960
	+ .043	- .032		+ 3.560	- 4.925
	+ .837	- 1.268	-32.263		- 45.221
	- 1.080	+ 2.076	+31.491	- 31.867	
		+ .187	- .364	+ .775	- 1.421
	- .237		+ .344	+ 1.486	- 3.453
	- .003	+ .002		- .208	+ .287
	+ .618	- .936	-23.814		- 33.377
	- .664	+ 1.277	+19.364	- 19.595	
		+ .072	- .140	+ .297	- .545
	- .102		+ .148	+ .639	- 1.486
	- .014	+ .011		- 1.163	+ 1.610
	+ .411	- .622	-15.833		- 22.191
	- .396	+ .760	+11.534	- 11.672	
		+ .025	- .049	+ .105	- .193
	- .043		+ .062	+ .269	- .624
	- .014	+ .010		- 1.123	+ 1.553
	+ .257	- .390	- 9.921		- 13.905
	- .231	+ .443	+ 6.717	- 6.797	
		+ .007	- .014	+ .031	- .056
	- .017		+ .025	+ .109	- .253
	- .010	+ .076		- .836	+ 1.157
	+ .155	- .235	- 5.985		- 8.390
	- .132	+ .254	+ 3.847	- 3.893	
		+ .001	- .021	+ .005	- .008
	- .007		+ .010	+ .042	- .097
	- .007	+ .005		- .558	+ .772
	+ .031	- .138	- 3.518		- 4.931
	- .075	+ .143	+ 2.175	- 2.201	
		- .001	+ .002	- .003	+ .006
	- .002		+ .003	+ .015	- .034
	- .004	+ .003		- .351	+ .485
	+ .053	- .080	- 2.029		- 2.844
	- .042	+ .080	+ 1.217	- 1.232	
		- .001	+ .002	- .004	+ .008
	- .001		+ .001	+ .004	- .010
	- .003	+ .002		- .212	+ .293
	+ .030	- .045			- 1.617
Σ =	+71.655	+30.793	+60.353	-195.786	-381.916

Fig. 7-2

Carry-Over Procedure

the final answers into the carry-over equations. Thus,

$$\overline{T}_1 = +\ 71.688 \doteq +\ 71.655$$

$$\overline{T}_2 = +\ 31.064 \doteq +\ 30.793$$

$$\overline{m}_1 = +\ 59.897 \doteq +\ 60.353$$

$$\overline{m}_2 = -\ 196.528 \doteq -\ 195.786$$

$$\overline{m}_3 = -\ 382.040 \doteq -\ 381.916 \ .$$

7-6 Comparison of Results

The final results are compared with those of Girkmann (10).

		Girkmann	
$\overline{T}_1$	+ 71.655	+ 71.584	kg/cm
$\overline{T}_2$	+ 30.793	+ 31.134	kg/cm
$\overline{m}_1$	+ 60.353	+ 61.09	cm.kg/cm
$\overline{m}_2$	- 195.786	- 199.08	cm.kg/cm
$\overline{m}_3$	- 381.916	- 387.02	cm.kg/cm

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8-1 Summary

The derivation of the three types of deformation equations for the folded plate structure and their solution by the carry-over method was presented in this thesis. By equating the longitudinal strains at a longitudinal joint, the Shear Flow Equation was formulated. The Three Moment Equation was obtained by summing the transverse angle changes at a joint to zero. From the general beam equations, the transverse plate displacements were expressed in terms of the transverse loads, moments, and longitudinal shears yielding the Displacement Equation.

To reduce the required number of equations, the Displacement Equations were substituted into the Three Moment Equation yielding the Seven Moment Equation. By eliminating the Displacement Equations, only the Shear Flow and Seven Moment Equations are required to solve the folded plate structure.

The main purpose of this thesis has been to extend the carry-over method to the solution of the folded plate equations. To solve the three sets of deformation equations, a one table carry-over with three sets of unknowns or a three table carry-over, each with one set of unknowns, was presented.

With the Seven Moment Equation, a one table carry-over with two sets of unknowns or a two table carry-over, each with one set of unknowns, was presented.

Also shown in this thesis were the particular deformation equations for the folded plate structure with vertical edge members. The plate theory was utilized for the thickened edge plate and beam theory was utilized for the edge member with a beam depth-to-thickness ratio.

8-2 Conclusions

The number of simultaneous equations necessary for analyzing a folded plate structure with n plates includes n-1 shear equations, n moment equations and n displacement equations. Thus, for a structure with a large number of plates, the solution becomes more difficult. Although the most expedient manner in which to solve the large number of simultaneous equations is the electronic computer, the carry-over method is a practical procedure when a computer is not available.

To reduce the number of simultaneous equations, the displacement equations can be eliminated by utilizing the Seven Moment Equation. Although the coefficients become more complex, many of the terms are repetitive and need be computed only once. The Seven Moment Equation can also be obtained by substituting the computed displacement coefficients into the Three Moment Equation. This has been demonstrated in the example problem.

In many structures, the displacement to moment carry-over factor is very large; thus, a small change in displacement is amplified many times in the moment. Because of this, it is sometimes difficult, if not impossible, to obtain convergency in the carry-over table which includes shears, moments, and displacements. For these cases it is convenient to obtain a solution by means of the Seven Moment Equation which has the displacement functions eliminated.

In general, when the exact solutions are desired, the one table carry-over procedure is the most efficient. When it becomes obvious that the displacement, moment, or shear has negligible influence on the results, an approximate solution may be obtained by means of the two or three table carry-over with the appropriate function neglected.

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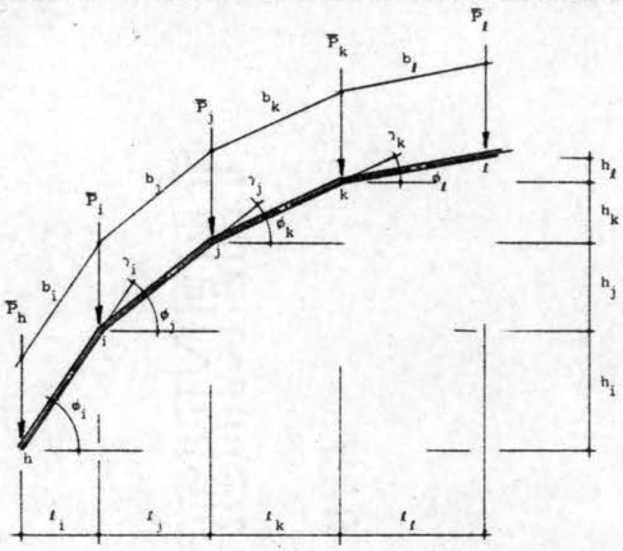
TABLE 1	SHEAR FLOW FUNCTIONS	TYPICAL PLATE



TYPICAL TRANSVERSE SECTION

SHEAR INFLUENCE COEFFICIENTS	
ΣQ_j	$+ 2(\frac{1}{A_j} + \frac{1}{A_k})$
Q_{ij}	$+ \frac{1}{A_j}$
Q_{kj}	$+ \frac{1}{A_k}$
MOMENT INFLUENCE COEFFICIENTS	
Q'_{hj}	$- \frac{3}{\sigma A_j b_j} (\frac{\cos\phi_i}{\ell_i \sin\gamma_i})$
Q'_{ij}	$+ \frac{3}{\sigma A_j b_j} \left[\frac{\cos\phi_k}{\ell_j \sin\gamma_j} + (\frac{1}{\ell_i} + \frac{1}{\ell_j}) \frac{\cos\phi_i}{\sin\gamma_i}\right] - \frac{3}{\sigma A_k b_k} (\frac{\cos\phi_j}{\ell_j \sin\gamma_j})$
Q'_{jj}	$- \frac{3}{\sigma A_j b_j} \left[\frac{\cos\phi_i}{\ell_j \sin\gamma_i} + (\frac{1}{\ell_j} + \frac{1}{\ell_k}) \frac{\cos\phi_k}{\sin\gamma_j}\right] + \frac{3}{\sigma A_k b_k} \left[\frac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + (\frac{1}{\ell_j} + \frac{1}{\ell_k}) \frac{\cos\phi_j}{\sin\gamma_j}\right]$
Q'_{kj}	$- \frac{3}{\sigma A_k b_k} \left[\frac{\cos\phi_j}{\ell_k \sin\gamma_j} + (\frac{1}{\ell_k} + \frac{1}{\ell_\ell}) \frac{\cos\phi_\ell}{\sin\gamma_k}\right] + \frac{3}{\sigma A_j b_j} (\frac{\cos\phi_k}{\ell_k \sin\gamma_j})$
$Q'_{\ell j}$	$+ \frac{3}{\sigma A_k b_k} (\frac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k})$
SHEAR LOAD COEFFICIENTS	
$\Sigma \bar{T}_j^*$	$+ \frac{3}{\sigma A_j b_j} (\frac{\cos\phi_k}{\sin\gamma_j} \bar{P}_j - \frac{\cos\phi_i}{\sin\gamma_i} \bar{P}_i) + \frac{3}{\sigma A_k b_k} (\frac{\cos\phi_\ell}{\sin\gamma_\ell} \bar{P}_k - \frac{\cos\phi_j}{\sin\gamma_j} \bar{P}_j)$

THREE SHEAR FLOW EQUATION

$$Q_{ij}\bar{T}_i + \Sigma Q_j \bar{T}_j + Q_{kj}\bar{T}_k = Q'_{hj}\bar{m}_h + Q'_{ij}\bar{m}_i + Q'_{jj}\bar{m}_j + Q'_{kj}\bar{m}_k + Q'_{\ell j}\bar{m}_\ell + \Sigma \bar{T}_j^*$$

CARRY-OVER EQUATION

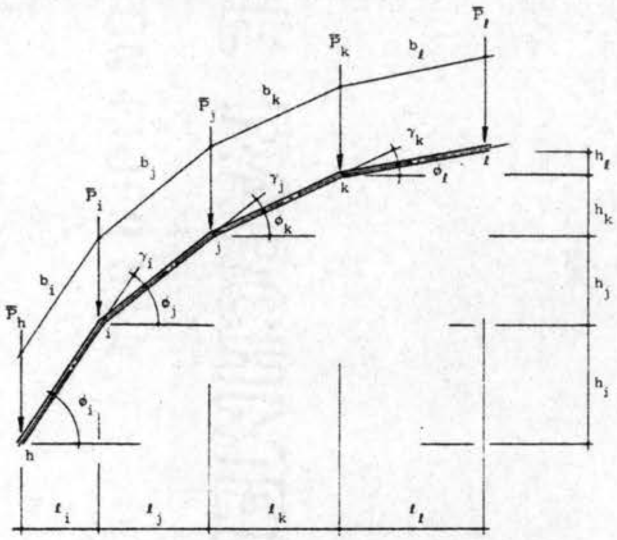
$$\bar{T}_j = + q_{ij}\bar{T}_i + q_{kj}\bar{T}_k + q'_{hj}\bar{m}_h + q'_{ij}\bar{m}_i + q'_{jj}\bar{m}_j + q'_{kj}\bar{m}_k + q'_{\ell j}\bar{m}_\ell + \bar{t}_j^*$$

EQUIVALENTS

$$q_{ij} = - \frac{Q_{ij}}{\Sigma Q_j} \qquad q'_{hj} = + \frac{Q'_{hj}}{\Sigma Q_j} \qquad q'_{jj} = + \frac{Q'_{jj}}{\Sigma Q_j} \qquad q'_{\ell j} = + \frac{Q_{\ell j}}{\Sigma Q_j}$$

$$q_{kj} = - \frac{Q_{kj}}{\Sigma Q_j} \qquad q_{ij} = + \frac{Q'_{ij}}{\Sigma Q_j} \qquad q'_{kj} = + \frac{Q'_{kj}}{\Sigma Q_j} \qquad \bar{t}_j^* = \frac{\Sigma \bar{T}_j^*}{\Sigma Q_j}$$

TABLE 2 DISPLACEMENT FUNCTIONS TYPICAL PLATE



TYPICAL TRANSVERSE SECTION

DISPLACEMENT INFLUENCE COEFFICIENT	
ΣR_j	$+ EI_j \alpha^4$
MOMENT INFLUENCE COEFFICIENTS	
R'_{hj}	$- \dfrac{\cos\phi_i}{\ell_i \sin\gamma_i}$
R'_{ij}	$+ \dfrac{\cos\phi_k}{\ell_j \sin\gamma_j} + \left(\dfrac{1}{\ell_i} + \dfrac{1}{\ell_j}\right) \dfrac{\cos\phi_i}{\sin\gamma_i}$
R'_{jj}	$- \dfrac{\cos\phi_i}{\ell_j \sin\gamma_i} - \left(\dfrac{1}{\ell_j} + \dfrac{1}{\ell_k}\right) \dfrac{\cos\phi_k}{\sin\gamma_j}$
R'_{kj}	$+ \dfrac{\cos\phi_k}{\ell_k \sin\gamma_j}$
SHEAR INFLUENCE COEFFICIENTS	
R''_{ij}	$- \dfrac{\alpha b_j}{2}$
R''_{jj}	$- \dfrac{\alpha b_j}{2}$
DISPLACEMENT LOAD COEFFICIENT	
$\Sigma \bar{V}^*_j$	$- \dfrac{\cos\phi_i}{\sin\gamma_i} \bar{P}_i + \dfrac{\cos\phi_k}{\sin\gamma_j} \bar{P}_j$

DISPLACEMENT EQUATION

$$\Sigma R_j \bar{v}_j = + R'_{hj}\bar{m}_h + R'_{ij}\bar{m}_i + R'_{jj}\bar{m}_j + R'_{kj}\bar{m}_k + R''_{ij}\bar{T}_i + R''_{jj}\bar{T}_j + \Sigma \bar{V}^*_j$$

CARRY-OVER EQUATION

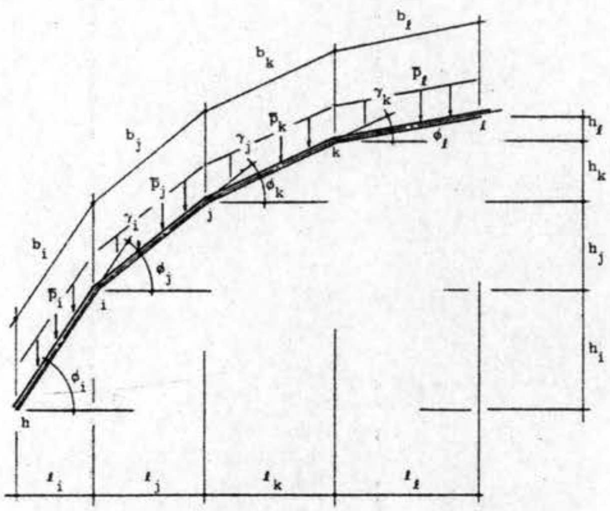
$$\bar{v}_j = + r'_{hj}\bar{m}_h + r'_{ij}\bar{m}_i + r'_{jj}\bar{m}_j + r'_{kj}\bar{m}_k + r''_{ij}\bar{T}_i + r''_{jj}\bar{T}_j + \bar{v}^*_j$$

EQUIVALENTS

$$r'_{hj} = + \frac{R'_{hj}}{\Sigma R_j} \qquad r'_{jj} = + \frac{R'_{jj}}{\Sigma R_j} \qquad r''_{ij} = + \frac{R''_{ij}}{\Sigma R_j} \qquad \bar{v}^*_j = + \frac{\Sigma \bar{V}^*_j}{\Sigma R_j}$$

$$r'_{ij} = + \frac{R'_{ij}}{\Sigma R_j} \qquad r'_{kj} = + \frac{R'_{kj}}{\Sigma R_j} \qquad r''_{jj} = + \frac{R''_{jj}}{\Sigma R_j}$$

TABLE 3 MOMENT FUNCTIONS TYPICAL PLATE



TYPICAL TRANSVERSE SECTION

MOMENT INFLUENCE COEFFICIENTS	
ΣF_j	$+\dfrac{b_j}{3EI'_j} + \dfrac{b_k}{3EI'_k}$
G_{ij}	$+\dfrac{b_j}{6EI'_j}$
G_{kj}	$+\dfrac{b_k}{6EI'_k}$
DISPLACEMENT INFLUENCE COEFFICIENTS	
G'_{ij}	$+\dfrac{1}{b_j \sin\gamma_i}$
G'_{jj}	$-\dfrac{1}{b_k \sin\gamma_j} - \dfrac{1}{b_j}(\cot\gamma_j + \cot\gamma_i)$
G'_{kj}	$+\dfrac{1}{b_j \sin\gamma_j} + \dfrac{1}{b_k}(\cot\gamma_k + \cot\gamma_j)$
$G'_{\ell j}$	$-\dfrac{1}{b_k \sin\gamma_k}$
MOMENT LOAD COEFFICIENTS	
$\Sigma\bar{\tau}_j^*$	$+\dfrac{\bar{p}_j b_j^2 \ell_j}{24EI'_j} + \dfrac{\bar{p}_k b_k^2 \ell_k}{24EI'_k}$

THREE MOMENT EQUATION

$$0 = G_{ij}\bar{m}_i + \Sigma F_j \bar{m}_j + G_{kj}\bar{m}_k + G'_{ij}\bar{v}_i + G'_{jj}\bar{v}_j + G'_{kj}\bar{v}_k$$
$$+ G'_{\ell j}\bar{v}_\ell + \Sigma\bar{\tau}_j^*$$

CARRY-OVER EQUATION

$$\bar{m}_j = g_{ij}\bar{m}_i + g_{kj}\bar{m}_k + g'_{ij}\bar{v}_i + g'_{jj}\bar{v}_j + g'_{kj}\bar{v}_k$$
$$+ g'_{\ell j}\bar{v}_\ell + \Sigma\bar{m}_j^*$$

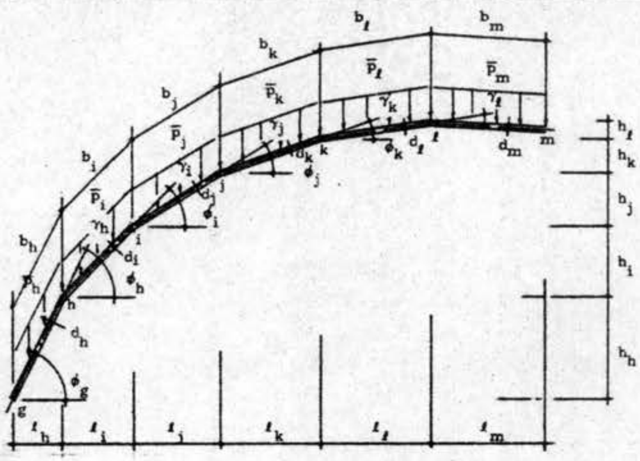
EQUIVALENTS

$$g_{ij} = -\frac{G_{ij}}{\Sigma F_j} \qquad g'_{ij} = -\frac{G'_{ij}}{\Sigma F_j} \qquad g'_{kj} = -\frac{G'_{kj}}{\Sigma F_j} \qquad \bar{m}_j^* = -\frac{\Sigma\bar{\tau}_j^*}{\Sigma F_j}$$

$$g_{kj} = -\frac{G_{kj}}{\Sigma F_j} \qquad g'_{jj} = -\frac{G'_{jj}}{\Sigma F_j} \qquad g'_{\ell j} = -\frac{G'_{\ell j}}{\Sigma F_j}$$

MOMENT FUNCTIONS

TYPICAL PLATE



TYPICAL TRANSVERSE SECTION

SEVEN MOMENT EQUATION

$$0 = \Sigma F_j \overline{m}_j + G_{gj}\overline{m}_g + G_{hj}\overline{m}_h + G_{ij}\overline{m}_i + G_{kj}\overline{m}_k + G_{\ell j}\overline{m}_\ell + G_{mj}\overline{m}_m + \Sigma\overline{\tau}_j$$

CARRY-OVER EQUATION

$$\overline{m}_j = + g_{gj}\overline{m}_g + g_{hj}\overline{m}_h + g_{ij}\overline{m}_i + g_{kj}\overline{m}_k + g_{\ell j}\overline{m}_\ell + g_{mj}\overline{m}_m$$

EQUIVALENTS

$$g_{gj} = -\frac{G_{gj}}{\Sigma F_j} \qquad g'_{hj} = -\frac{G'_{hj}}{\Sigma F_j} \qquad \overline{S}_{ij} = \frac{\cos\phi_j}{\sin\gamma_j} \qquad \overline{\Delta}_{ij} = \frac{\cos\phi_j}{\sin\gamma_j}\,\frac{1}{EI_j\sigma^4}$$

$$g_{hj} = -\frac{G_{hj}}{\Sigma F_j} \qquad g'_{ij} = -\frac{G'_{ij}}{\Sigma F_j} \qquad \overline{S}_{ji} = \frac{\cos\phi_k}{\sin\gamma_j} \qquad \overline{\Delta}_{ji} = \frac{\cos\phi_k}{\sin\gamma_j}\,\frac{1}{EI_j\sigma^4}$$

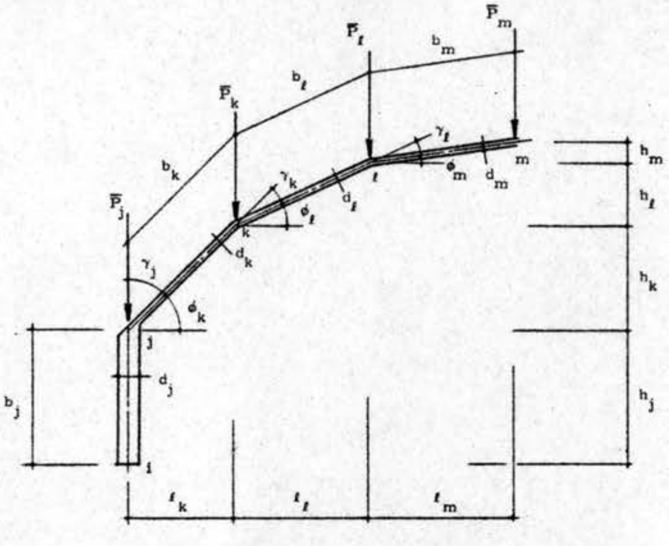
$$g_{ij} = -\frac{G_{ij}}{\Sigma F_j} \qquad g'_{jj} = -\frac{G'_{jj}}{\Sigma F_j} \qquad \overline{S}_{jk} = \frac{\cos\phi_k}{\sin\gamma_k} \qquad \overline{\Delta}_{jk} = \frac{\cos\phi_k}{\sin\gamma_k}\,\frac{1}{EI_k\sigma^4}$$

$$g_{kj} = -\frac{G_{kj}}{\Sigma F_j} \qquad g'_{kj} = -\frac{G'_{kj}}{\Sigma F_j} \qquad \overline{S}_{kj} = \frac{\cos\phi_\ell}{\sin\gamma_k} \qquad \overline{\Delta}_{kj} = \frac{\cos\phi_\ell}{\sin\gamma_k}\,\frac{1}{EI_k\sigma^4}$$

$$g_{\ell j} = -\frac{G_{\ell j}}{\Sigma F_j} \qquad g'_{\ell j} = -\frac{G'_{\ell j}}{\Sigma F_j} \qquad \overline{\Delta}'_{ij} = \overline{\Delta}'_{ji} = \frac{b_j}{2EI'_j\sigma^3}$$

$$g_{mj} = -\frac{G_{mj}}{\Sigma F_j} \qquad \overline{\Delta}'_{jk} = \overline{\Delta}'_{kj} = \frac{b_k}{2EI'_k\sigma^3}$$

MOMENT INFLUENCE COEFFICIENTS	
F_{ji}	$+\frac{b_j}{3EI'_j} + \frac{1}{\ell_j b_j}\left[\overline{\Delta}_{ij}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{ih}}{\sin\gamma_i}\right] + \left(\frac{1}{\ell_j} + \frac{1}{\ell_k}\right)\frac{1}{b_j}\left[\overline{\Delta}_{ji}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{jk}}{\sin\gamma_j}\right] + \frac{1}{\ell_k b_j}\,\frac{\overline{\Delta}_{kj}}{\sin\gamma_j}$
F_{jk}	$+\frac{b_k}{3EI'_k} + \frac{1}{\ell_k b_k}\left[\overline{\Delta}_{kj}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{k\ell}}{\sin\gamma_k}\right] + \left(\frac{1}{\ell_j} + \frac{1}{\ell_k}\right)\frac{1}{b_k}\left[\overline{\Delta}_{jk}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{ji}}{\sin\gamma_j}\right] + \frac{1}{\ell_j b_k}\,\frac{\overline{\Delta}_{ij}}{\sin\gamma_j}$
G_{gj}	$-\frac{1}{b_h \ell_j}\,\frac{\overline{\Delta}_{hi}}{\sin\gamma_i}$
G_{hj}	$+\left(\frac{1}{\ell_h} + \frac{1}{\ell_i}\right)\frac{1}{b_j}\,\frac{\overline{\Delta}_{hi}}{\sin\gamma_i} + \frac{1}{\ell_i b_j}\left[\overline{\Delta}_{ij}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{ih}}{\sin\gamma_i}\right] + \frac{1}{\ell_i b_k}\,\frac{\overline{\Delta}_{ij}}{\sin\gamma_j}$
G_{ij}	$+\frac{b_j}{6EI'_j} - \frac{1}{\ell_i b_j}\,\frac{\overline{\Delta}_{hi}}{\sin\gamma_i} - \left(\frac{1}{\ell_i} + \frac{1}{\ell_j}\right)\frac{1}{b_j}\left[\overline{\Delta}_{ij}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{ih}}{\sin\gamma_i}\right] - \left(\frac{1}{\ell_i} + \frac{1}{\ell_j}\right)\frac{1}{b_k}\,\frac{\overline{\Delta}_{ij}}{\sin\gamma_j}$ $-\frac{1}{\ell_j b_j}\left[\overline{\Delta}_{ji}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{jk}}{\sin\gamma_j}\right] - \frac{1}{\ell_j b_k}\left[\overline{\Delta}_{jk}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{ji}}{\sin\gamma_j}\right]$
G_{kj}	$+\frac{b_k}{6EI'_k} - \frac{1}{\ell_k b_k}\,\frac{\overline{\Delta}_{\ell k}}{\sin\gamma_k} - \left(\frac{1}{\ell_k} + \frac{1}{\ell_\ell}\right)\frac{1}{b_k}\left[\overline{\Delta}_{kj}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{k\ell}}{\sin\gamma_k}\right] - \left(\frac{1}{\ell_k} + \frac{1}{\ell_\ell}\right)\frac{1}{b_j}\,\frac{\overline{\Delta}_{kj}}{\sin\gamma_j}$ $-\frac{1}{\ell_k b_k}\left[\overline{\Delta}_{jk}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{ji}}{\sin\gamma_j}\right] - \frac{1}{\ell_k b_j}\left[\overline{\Delta}_{ji}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{\Delta}_{jk}}{\sin\gamma_j}\right]$
$G_{\ell j}$	$+\left(\frac{1}{\ell_\ell} + \frac{1}{\ell_m}\right)\frac{1}{b_k}\,\frac{\overline{\Delta}_{\ell k}}{\sin\gamma_k} + \frac{1}{\ell_\ell b_k}\left[\overline{\Delta}_{kj}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{\Delta}_{k\ell}}{\sin\gamma_k}\right] + \frac{1}{\ell_\ell b_j}\,\frac{\overline{\Delta}_{kj}}{\sin\gamma_j}$
G_{mj}	$-\frac{1}{\ell_m b_k}\,\frac{\overline{\Delta}_{\ell k}}{\sin\gamma_k}$
SHEAR INFLUENCE COEFFICIENTS	
G'_{hj}	$-\frac{1}{b_j}\,\frac{\overline{\Delta}'_{hi}}{\sin\gamma_i}$
G'_{ij}	$+\frac{1}{b_j}\left[\overline{\Delta}'_{ij}(\cot\gamma_i + \cot\gamma_j) - \frac{\overline{\Delta}'_{ih}}{\sin\gamma_i}\right] + \frac{1}{b_k}\,\frac{\overline{\Delta}'_{ij}}{\sin\gamma_j}$
G'_{jj}	$+\frac{1}{b_j}\left[\overline{\Delta}'_{ji}(\cot\gamma_i + \cot\gamma_j) - \frac{\overline{\Delta}'_{jk}}{\sin\gamma_j}\right] - \frac{1}{b_k}\left[\overline{\Delta}'_{jk}(\cot\gamma_j + \cot\gamma_k) - \frac{\overline{\Delta}'_{ji}}{\sin\gamma_j}\right]$
G'_{kj}	$-\frac{1}{b_k}\left[\overline{\Delta}'_{kj}(\cot\gamma_j + \cot\gamma_k) - \frac{\overline{\Delta}'_{k\ell}}{\sin\gamma_k}\right] - \frac{1}{b_j}\,\frac{\overline{\Delta}'_{kj}}{\sin\gamma_j}$
$G'_{\ell j}$	$+\frac{1}{b_k}\,\frac{\overline{\Delta}'_{\ell k}}{\sin\gamma_k}$
MOMENT LOAD COEFFICIENTS	
$\Sigma\overline{\tau}^*_j$	$+\frac{\overline{p}_j b_j^2 \ell_j}{24EI_j} + \frac{\overline{p}_k b_k^2 \ell_k}{24EI_k} - \frac{1}{b_j}\,\frac{\overline{S}_{hi}}{\sin\gamma_i} + \frac{1}{b_j}\left[\overline{S}_{ij}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{S}_{ih}}{\sin\gamma_i}\right] + \frac{1}{b_k}\,\frac{\overline{S}_{ij}}{\sin\gamma_j} - \frac{1}{b_j}\left[\overline{S}_{ji}(\cot\gamma_i + \cot\gamma_j) + \frac{\overline{S}_{jk}}{\sin\gamma_j}\right]$ $-\frac{1}{b_k}\left[\overline{S}_{jk}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{S}_{ji}}{\sin\gamma_j}\right] + \frac{1}{b_k}\left[\overline{S}_{kj}(\cot\gamma_j + \cot\gamma_k) + \frac{\overline{S}_{k\ell}}{\sin\gamma_k}\right] + \frac{1}{b_j}\,\frac{\overline{S}_{kj}}{\sin\gamma_j} - \frac{1}{b_k}\,\frac{\overline{S}_{\ell k}}{\sin\gamma_k}$



TRANSVERSE SECTION

THREE SHEAR FLOW EQUATIONS

$$\Sigma Q_j \bar{T}_j + Q_{kj}\bar{T}_k = + Q'_{jj}\bar{m}_j + Q'_{kj}\bar{m}_k + Q'_{\ell j}\bar{m}_\ell + Q''_{jj}\bar{v}_j + Q''_{kj}\bar{v}_k + \Sigma\bar{T}^*_j$$

$$Q_{jk}\bar{T}_j + \Sigma Q_k \bar{T}_k + Q_{\ell k}\bar{T}_\ell = Q'_{jk}\bar{m}_j + Q'_{kk}\bar{m}_k + Q'_{\ell k}\bar{m}_\ell + Q'_{mk}\bar{m}_m + Q''_{jk}\bar{v}_j + Q''_{kk}\bar{v}_k + \Sigma\bar{T}^*_k$$

CARRY-OVER EQUATIONS

$$\bar{T}_j = + q_{kj}\bar{T}_k + q'_{jj}\bar{m}_j + q'_{kj}\bar{m}_k + q'_{\ell j}\bar{m}_\ell + q''_{jj}\bar{v}_j + q''_{kj}\bar{v}_k + \bar{t}^*_j$$

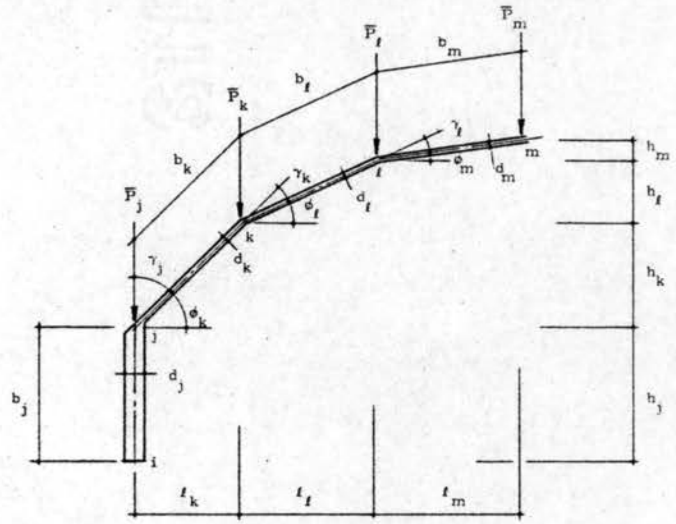
$$\bar{T}_k = + q_{jk}\bar{T}_j + q_{\ell k}\bar{T}_\ell + q'_{jk}\bar{m}_j + q'_{kk}\bar{m}_k + q'_{\ell k}\bar{m}_\ell + q'_{mk}\bar{m}_m + q''_{jk}\bar{v}_j + q''_{kk}\bar{v}_k + \bar{t}^*_k$$

EQUIVALENTS

$q_{kj} = -\dfrac{Q_{kj}}{\Sigma Q_j}$	$q''_{jj} = +\dfrac{Q''_{jj}}{\Sigma Q_j}$	$q_{\ell k} = -\dfrac{Q_{\ell k}}{\Sigma Q_k}$	$q'_{mk} = +\dfrac{Q'_{mk}}{\Sigma Q_k}$
$q'_{jj} = +\dfrac{Q'_{jj}}{\Sigma Q_j}$	$q''_{kj} = +\dfrac{Q''_{kj}}{\Sigma Q_j}$	$q'_{jk} = +\dfrac{Q'_{jk}}{\Sigma Q_k}$	$q''_{jk} = +\dfrac{Q''_{jk}}{\Sigma Q_k}$
$q'_{kj} = +\dfrac{Q'_{kj}}{\Sigma Q_j}$	$\bar{t}^*_j = +\dfrac{\Sigma\bar{T}^*_j}{\Sigma Q_j}$	$q'_{kk} = +\dfrac{Q'_{kk}}{\Sigma Q_k}$	$q''_{kk} = +\dfrac{Q''_{kk}}{\Sigma Q_k}$
$q'_{\ell j} = +\dfrac{Q'_{\ell j}}{\Sigma Q_j}$	$q_{jk} = -\dfrac{Q_{jk}}{\Sigma Q_k}$	$q'_{\ell k} = +\dfrac{Q'_{\ell k}}{\Sigma Q_k}$	$\bar{t}^*_k = +\dfrac{\Sigma\bar{T}^*_k}{\Sigma Q_k}$

JOINT j	
SHEAR INFLUENCE COEFFICIENTS	
ΣQ_j	$+2\left(\dfrac{1}{A_j} + \dfrac{1}{A_k}\right)$
Q_{kj}	$+\dfrac{1}{A_k}$
MOMENT INFLUENCE COEFFICIENTS	
Q'_{jj}	$-\dfrac{3}{\alpha A_j b_j}\left(\dfrac{1}{\ell_k} + \zeta_j \tan\phi_k\right)$ $+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \dfrac{\zeta_j}{\cos\phi_k}\right)$
Q'_{kj}	$+\dfrac{3}{\alpha A_j b_j}\left(\dfrac{1}{\ell_k}\right)$ $-\dfrac{3}{\alpha A_k b_k}\left(\dfrac{1}{\ell_k} + \dfrac{1}{\ell_\ell}\right)\dfrac{\cos\phi_\ell}{\sin\gamma_k}$
$Q_{\ell j}$	$+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k}\right)$
DISPLACEMENT INFLUENCE COEFFICIENTS	
Q''_{jj}	$-\dfrac{3}{\alpha A_j b_j}\left(\zeta'_j \cot\gamma_j \tan\phi_k\right)$ $+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\zeta'_j \cot\gamma_j}{\cos\phi_k}\right)$
Q''_{kj}	$+\dfrac{3}{\alpha A_j b_j}\left(\zeta'_j \dfrac{\tan\phi_k}{\sin\gamma_j}\right)$ $-\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\zeta'_j}{\sin\gamma_j \cos\phi_k}\right)$
SHEAR LOAD COEFFICIENTS	
$\Sigma\bar{T}^*_j$	$+\dfrac{3}{\alpha A_j b_j}(\bar{P}_j) + \dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\sin\gamma_k}\bar{P}_k\right)$

JOINT k	
SHEAR INFLUENCE COEFFICIENTS	
ΣQ_k	$+2\left(\dfrac{1}{A_k} + \dfrac{1}{A_\ell}\right)$
Q_{jk}	$+\dfrac{1}{A_k}$
$Q_{\ell k}$	$+\dfrac{1}{A_\ell}$
MOMENT INFLUENCE COEFFICIENTS	
Q'_{jk}	$+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \dfrac{\zeta_j}{\cos\phi_k}\right)$ $-\dfrac{3}{\alpha A_\ell b_\ell}\left(\dfrac{\cos\phi_k}{\ell_k \sin\gamma_k}\right)$
Q'_{kk}	$-\dfrac{3}{\alpha A_k b_k}\left(\dfrac{1}{\ell_k} + \dfrac{1}{\ell_\ell}\right)\dfrac{\cos\phi_\ell}{\sin\gamma_k}$ $+\dfrac{3}{\alpha A_\ell b_\ell}\left[\dfrac{\cos\phi_m}{\ell_\ell \sin\gamma_\ell} + \left(\dfrac{1}{\ell_k} + \dfrac{1}{\ell_\ell}\right)\dfrac{\cos\phi_k}{\sin\gamma_k}\right]$
$Q'_{\ell k}$	$-\dfrac{3}{\alpha A_\ell b_\ell}\left[\dfrac{\cos\phi_k}{\ell_\ell \sin\gamma_k} + \left(\dfrac{1}{\ell_\ell} + \dfrac{1}{\ell_m}\right)\dfrac{\cos\phi_m}{\sin\gamma_\ell}\right]$ $+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k}\right)$
Q'_{mk}	$+\dfrac{3}{\alpha A_\ell b_\ell}\left(\dfrac{\cos\phi_m}{\ell_m \sin\gamma_\ell}\right)$
DISPLACEMENT INFLUENCE COEFFICIENTS	
Q''_{jk}	$+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\zeta'_j \cot\gamma_j}{\cos\phi_k}\right)$
Q''_{kk}	$-\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\zeta'_j}{\sin\gamma_j \cos\phi_k}\right)$
SHEAR LOAD COEFFICIENTS	
$\Sigma\bar{T}^*_k$	$+\dfrac{3}{\alpha A_k b_k}\left(\dfrac{\cos\phi_\ell}{\sin\gamma_k}\bar{P}_k\right)$ $+\dfrac{3}{\alpha A_\ell b_\ell}\left(\dfrac{\cos\phi_m}{\sin\gamma_\ell}\bar{P}_\ell - \dfrac{\cos\phi_k}{\sin\gamma_k}\bar{P}_k\right)$



TRANSVERSE SECTION

DISPLACEMENT EQUATIONS

$$\Sigma R_j \bar{v}_j = + R_{kj}\bar{v}_k + R'_{jj}\overline{m}_j + R'_{kj}\overline{m}_k + R''_{jj}\bar{T}_j + \Sigma\bar{V}^*_j$$

$$\Sigma R_k \bar{v}_k = + R_{jk}\bar{v}_j + R'_{jk}\overline{m}_j + R'_{kk}\overline{m}_k + R'_{\ell k}\overline{m}_k + R''_{jk}\bar{T}_j + R''_{kk}\bar{T}_k + \Sigma\bar{V}^*_k$$

CARRY-OVER EQUATIONS

$$\bar{v}_j = + r_{kj}\bar{v}_k + r'_{jj}\overline{m}_j + r'_{kj}\overline{m}_k + r''_{jj}\bar{T}_j + \bar{v}^*_j$$

$$\bar{v}_k = + r_{jk}\bar{v}_j + r'_{jk}\overline{m}_j + r'_{kk}\bar{v}_k + r'_{\ell k}\bar{v}_\ell + r''_{jk}\bar{T}_j + r''_{kk}\bar{T}_k + \bar{v}^*_k$$

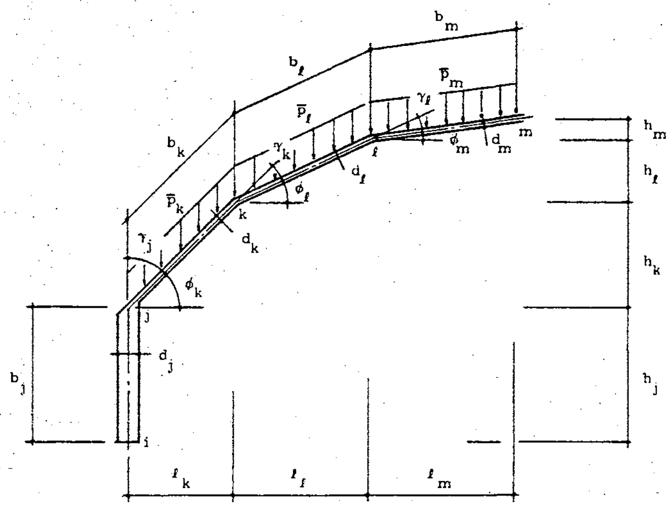
EQUIVALENTS

$$r_{kj} = + \frac{R_{kj}}{\Sigma R_j} \qquad r'_{kj} = + \frac{R'_{kj}}{\Sigma R_j} \qquad \bar{v}^*_j = + \frac{\Sigma\bar{V}^*_j}{\Sigma R_j} \qquad r'_{jk} = + \frac{R'_{jk}}{\Sigma R_k} \qquad r'_{\ell k} = + \frac{R'_{\ell k}}{\Sigma R_k} \qquad r''_{kk} = + \frac{R''_{kk}}{\Sigma R_k}$$

$$r'_{jj} = + \frac{R'_{jj}}{\Sigma R_j} \qquad r''_{jj} = + \frac{R''_{jj}}{\Sigma R_j} \qquad r_{kj} = + \frac{R_{kj}}{\Sigma R_k} \qquad r'_{kk} = + \frac{R'_{kk}}{\Sigma R_k} \qquad r''_{jk} = + \frac{R''_{jk}}{\Sigma R_k} \qquad \bar{v}^*_k = + \frac{\Sigma\bar{V}^*_k}{\Sigma R_k}$$

PLATE j	
DISPLACEMENT INFLUENCE COEFFICIENTS	
ΣR_j	$+ EI_j\alpha^4 + \zeta'_j \cot\gamma_j \tan\phi_k$
R_{kj}	$+ \dfrac{\zeta'_j \tan\phi_k}{\sin\gamma_j}$
MOMENT INFLUENCE COEFFICIENTS	
R'_{jj}	$- (\dfrac{1}{\ell_k} + \zeta_j \tan\phi_k)$
R'_{kj}	$+ \dfrac{1}{\ell_k}$
SHEAR INFLUENCE COEFFICIENTS	
R''_{jj}	$- \alpha \dfrac{b_j}{2}$
DISPLACEMENT LOAD COEFFICIENT	
$\Sigma\bar{V}^*_j$	$+ \bar{P}_j$

PLATE k	
DISPLACEMENT INFLUENCE COEFFICIENTS	
ΣR_k	$+ EI_k\alpha^4 + \dfrac{\zeta'_j}{\sin\gamma_j \cos\phi_k}$
R_{jk}	$+ \dfrac{\zeta'_j \cot\gamma_j}{\cos\phi_k}$
MOMENT INFLUENCE COEFFICIENTS	
R'_{jk}	$+ \dfrac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \dfrac{\zeta_j}{\cos\phi_k}$
R'_{kk}	$- (\dfrac{1}{\ell_k} + \dfrac{1}{\ell_\ell}) \dfrac{\cos\phi_\ell}{\sin\gamma_k}$
$R_{\ell k}$	$+ \dfrac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k}$
SHEAR INFLUENCE COEFFICIENTS	
R''_{jk}	$- \alpha \dfrac{b_k}{2}$
R''_{kk}	$- \alpha \dfrac{b_k}{2}$
DISPLACEMENT LOAD COEFFICIENT	
$\Sigma\bar{V}^*_k$	$+ \dfrac{\cos\phi_\ell}{\sin\gamma_k} \bar{P}_k$



TRANSVERSE SECTION

MOMENT INFLUENCE COEFFICIENTS	
ΣF_j	$+ \; X_j + \dfrac{b_k}{3EI'_k}$
G_{kj}	$+ \; \dfrac{b_k}{6EI'_k}$
DISPLACEMENT INFLUENCE COEFFICIENTS	
G'_{jj}	$- \; \zeta_j(\cot\gamma_j) - \dfrac{1}{b_k}\left(\dfrac{1}{\sin\gamma_j}\right)$
G'_{kj}	$+ \; \zeta_j\left(\dfrac{1}{\sin\gamma_j}\right) + \dfrac{1}{b_k}(\cot\gamma_k + \cot\gamma_j)$
$G'_{\ell j}$	$- \; \dfrac{1}{b_k}\left(\dfrac{1}{\sin\gamma_k}\right)$
MOMENT LOAD COEFFICIENTS	
$\Sigma\bar{\tau}^*_j$	$+ \; \dfrac{\bar{p}_k b_k^2 \ell_k}{24\,EI'_k}$

THREE MOMENT EQUATION

$$O = \Sigma F_j \bar{m}_j + G_{kj}\bar{m}_k + G'_{jj}\bar{v}_j + G'_{kj}\bar{v}_k + G'_{\ell j}\bar{v}_\ell + \Sigma\bar{\tau}^*_j$$

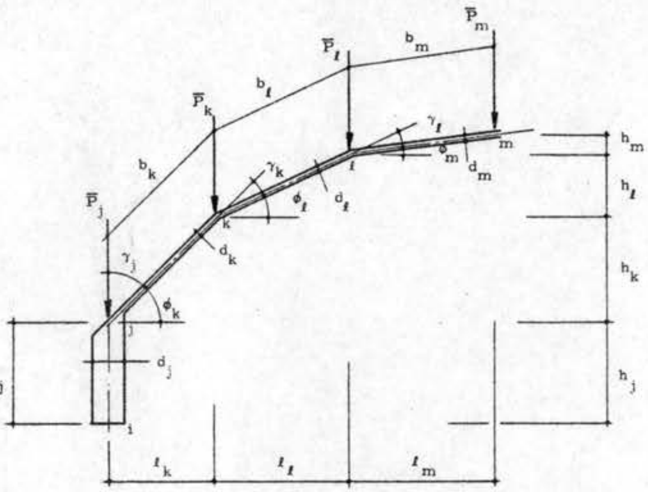
CARRY-OVER EQUATION

$$\bar{m}_j = + \; g_{kj}\bar{m}_k + g'_{jj}\bar{v}_j + g'_{kj}\bar{v}_k + g'_{\ell j}\bar{v}_\ell + \bar{m}^*_j$$

EQUIVALENTS

$$g_{kj} = -\frac{G_{kj}}{\Sigma F_j} \qquad g'_{kj} = -\frac{G'_{kj}}{\Sigma F_j} \qquad g'_{\ell j} = -\frac{G'_{\ell j}}{\Sigma F_j}$$

$$g'_{jj} = -\frac{G'_{jj}}{\Sigma F_j} \qquad g'_{\ell j} = -\frac{G'_{\ell j}}{\Sigma F_j} \qquad \bar{m}^*_j = -\frac{\Sigma\bar{\tau}^*_j}{\Sigma F_j}$$



TRANSVERSE SECTION

THREE SHEAR FLOW EQUATIONS

$$\Sigma Q_j \bar{T}_j + Q_{kj}\bar{T}_k = + Q'_{jj}\bar{m}_j + Q'_{kj}\bar{m}_k + Q'_{\ell j}\bar{m}_\ell + Q''_{jj}\bar{v}_j + Q''_{kj}\bar{v}_k + \Sigma\bar{T}^*_j$$

$$Q_{jk}\bar{T}_j + \Sigma Q_k \bar{T}_k + Q_{\ell k}\bar{T}_\ell = Q'_{jk}\bar{m}_j + Q'_{kk}\bar{m}_k + Q'_{\ell k}\bar{m}_\ell + Q'_{mk}\bar{m}_m + Q''_{jk}\bar{v}_j + Q''_{kk}v_k + \Sigma\bar{T}^*_k$$

CARRY-OVER EQUATIONS

$$\bar{T}_j = + q_{kj}\bar{T}_k + q'_{jj}\bar{m}_j + q'_{kj}\bar{m}_k + q'_{\ell j}\bar{m}_\ell + q''_{jj}\bar{v}_j + q''_{kj}\bar{v}_k + \bar{t}^*_j$$

$$\bar{T}_k = + q_{jk}\bar{T}_j + q_{\ell k}\bar{T}_\ell + q'_{jk}\bar{m}_j + q'_{kk}\bar{m}_k + q'_{\ell k}\bar{m}_\ell + q'_{mk}\bar{m}_m + q''_{jk}\bar{v}_j + q''_{kk}v_k + \bar{t}^*_k$$

EQUIVALENTS

$$q_{kj} = -\frac{Q_{kj}}{\Sigma Q_j} \qquad q''_{jj} = +\frac{Q''_{jj}}{\Sigma Q_j} \qquad q_{\ell k} = -\frac{Q_{\ell k}}{\Sigma Q_k} \qquad q'_{mk} = +\frac{Q'_{mk}}{\Sigma Q_k}$$

$$q'_{jj} = +\frac{Q'_{jj}}{\Sigma Q_j} \qquad q''_{kj} = +\frac{Q''_{kj}}{\Sigma Q_j} \qquad q'_{jk} = +\frac{Q'_{jk}}{\Sigma Q_k} \qquad q''_{jk} = +\frac{Q''_{jk}}{\Sigma Q_k}$$

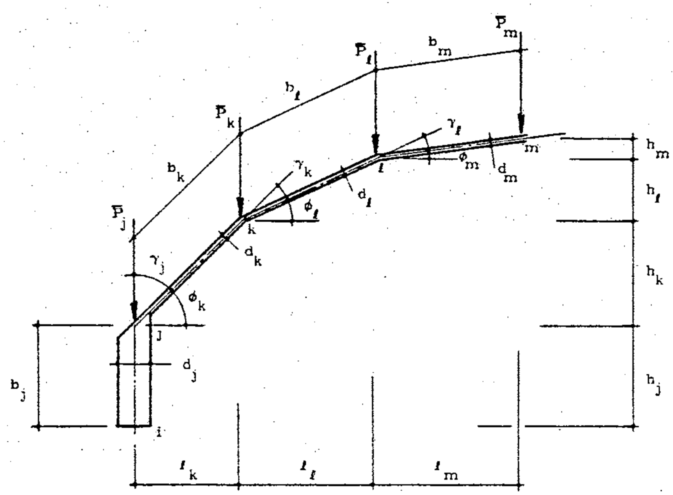
$$q'_{kj} = +\frac{Q'_{kj}}{\Sigma Q_j} \qquad \bar{t}^*_j = +\frac{\Sigma\bar{T}^*_j}{\Sigma Q_j} \qquad q'_{kk} = +\frac{Q'_{kk}}{\Sigma Q_k} \qquad q''_{kk} = +\frac{Q''_{kk}}{\Sigma Q_k}$$

$$q'_{\ell j} = +\frac{Q'_{\ell j}}{\Sigma Q_j} \qquad q_{jk} = -\frac{Q_{jk}}{\Sigma Q_k} \qquad q'_{\ell k} = +\frac{Q'_{\ell k}}{\Sigma Q_k} \qquad \bar{t}^*_k = +\frac{\Sigma\bar{T}^*_k}{\Sigma Q_k}$$

JOINT j	
SHEAR INFLUENCE COEFFICIENTS	
ΣQ_j	$+2(\frac{1}{A_j} + \frac{1}{A_k})$
Q_{kj}	$+\frac{1}{A_k}$
MOMENT INFLUENCE COEFFICIENTS	
Q''_{jj}	$-\frac{3}{\sigma A_j b_j}(\frac{1}{\ell_k} + \frac{2}{b_j}\tan\phi_k)$ $+\frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \frac{2}{b_j \cos\phi_k})$
Q'_{kj}	$+\frac{3}{\sigma A_j b_j}(\frac{1}{\ell_k})$ $-\frac{3}{\sigma A_k b_k}(\frac{1}{\ell_k} + \frac{1}{\ell_\ell})\frac{\cos\phi_\ell}{\sin\gamma_k}$
$Q'_{\ell j}$	$+\frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k})$
DISPLACEMENT INFLUENCE COEFFICIENTS	
Q''_{jj}	$-\frac{3}{\sigma A_j b_j}\frac{(EI_{jy}\sigma^4)^2\cot\gamma_j\tan\phi_k}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}$ $+\frac{3}{\sigma A_k b_k}\frac{(EI_{jy}\sigma^4)^2}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}(\frac{\cot\gamma_j}{\cos\phi_k})$
Q''_{kj}	$+\frac{3}{\sigma A_j b_j}\frac{(EI_{jy}\sigma^4)^2}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}(\frac{\tan\phi_k}{\sin\gamma_j})$ $-\frac{3}{\sigma A_k b_k}\frac{(EI_{jy}\sigma^4)^2}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}(\frac{1}{\sin\gamma_j \cos\phi_k})$
SHEAR LOAD COEFFICIENTS	
$\Sigma\bar{T}^*_j$	$+\frac{3}{\sigma A_j b_j}(\bar{P}_j) + \frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\sin\gamma_k}\bar{P}_k)$

JOINT k	
SHEAR INFLUENCE COEFFICIENTS	
ΣQ_k	$+2(\frac{1}{A_k} + \frac{1}{A_\ell})$
Q_{jk}	$+\frac{1}{A_k}$
$Q_{\ell k}$	$+\frac{1}{A_\ell}$
MOMENT INFLUENCE COEFFICIENTS	
Q'_{jk}	$+\frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \frac{2}{b_j \cos\phi_k})$ $-\frac{3}{\sigma A_\ell b_\ell}(\frac{\cos\phi_k}{\ell_k \sin\gamma_k})$
Q'_{kk}	$-\frac{3}{\sigma A_k b_k}(\frac{1}{\ell_k} + \frac{1}{\ell_\ell})\frac{\cos\phi_\ell}{\sin\gamma_k}$ $+\frac{3}{\sigma A_\ell b_\ell}\left[\frac{\cos\phi_m}{\ell_\ell \sin\gamma_\ell} + (\frac{1}{\ell_k} + \frac{1}{\ell_\ell})\frac{\cos\phi_k}{\sin\gamma_k}\right]$
$Q'_{\ell k}$	$-\frac{3}{\sigma A_\ell b_\ell}\left[\frac{\cos\phi_k}{\ell_\ell \sin\gamma_k} + (\frac{1}{\ell_\ell} + \frac{1}{\ell_m})\frac{\cos\phi_m}{\sin\gamma_\ell}\right]$ $+\frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k})$
Q'_{mk}	$+\frac{3}{\sigma A_\ell b_\ell}(\frac{\cos\phi_m}{\ell_m \sin\gamma_\ell})$
DISPLACEMENT INFLUENCE COEFFICIENTS	
Q''_{jk}	$+\frac{3}{\sigma A_k b_k}\frac{(EI_{jy}\sigma^4)^2}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}(\frac{\cot\gamma_j}{\cos\phi_k})$
Q''_{kk}	$-\frac{3}{\sigma A_k b_k}\frac{(EI_{jy}\sigma^4)^2}{(4JG\sigma^2 + b_j^2 EI_{jy}\sigma^4)}(\frac{1}{\sin\gamma_j \cos\phi_k})$
SHEAR LOAD COEFFICIENTS	
$\Sigma\bar{T}^*_k$	$+\frac{3}{\sigma A_k b_k}(\frac{\cos\phi_\ell}{\sin\gamma_k}\bar{P}_k)$ $+\frac{3}{\sigma A_\ell b_\ell}(\frac{\cos\phi_m}{\sin\gamma_\ell}\bar{P}_\ell - \frac{\cos\phi_k}{\sin\gamma_k}\bar{P}_k)$

TABLE 9 DISPLACEMENT FUNCTIONS EDGE BEAM



TRANSVERSE SECTION

PLATE j	
DISPLACEMENT INFLUENCE COEFFICIENTS	
ΣR_j	$+ EI_j\alpha^4 + \frac{(EI_{jy}\alpha^4)^2 \cot\gamma_j \tan\phi_k}{(4JG\alpha^2 + b_j^2 EI_{jy}\alpha^4)}$
R_{kj}	$+ \frac{(EI_{jy}\alpha^4)^2}{(4JG\alpha^2 + b_j^2 EI_{jy}\alpha^4)} \left(\frac{\tan\phi_k}{\sin\gamma_j}\right)$
MOMENT INFLUENCE COEFFICIENTS	
R'_{jj}	$-\left(\frac{1}{\ell_k} + \frac{2}{b_j}\tan\phi_k\right)$
R'_{kj}	$+\frac{1}{\ell_k}$
SHEAR INFLUENCE COEFFICIENTS	
R''_{jj}	$-\alpha\frac{b_j}{2}$
DISPLACEMENT LOAD COEFFICIENT	
$\Sigma \bar{V}_j^*$	$+\bar{P}_j$

PLATE k	
DISPLACEMENT INFLUENCE COEFFICIENTS	
ΣR_k	$+ EI_k\alpha^4 \frac{(EI_{jy}\alpha^4)^2}{(4JG\alpha^2 + b_j^2 EI_{jy}\alpha^4)} \left(\frac{1}{\sin\gamma_j \cos\phi_k}\right)$
R_{jk}	$+ \frac{(EI_{jy}\alpha^4)^2}{(4JG\alpha^2 + b_j^2 EI_{jy}\alpha^4)} \left(\frac{\cot\gamma_j}{\cos\phi_k}\right)$
MOMENT INFLUENCE COEFFICIENTS	
R'_{jk}	$+\frac{\cos\phi_\ell}{\ell_k \sin\gamma_k} + \frac{2}{b_j \cos\phi_k}$
R'_{kk}	$-\left(\frac{1}{\ell_k} + \frac{1}{\ell_\ell}\right)\frac{\cos\phi_\ell}{\sin\gamma_k}$
$R'_{\ell k}$	$+\frac{\cos\phi_\ell}{\ell_\ell \sin\gamma_k}$
SHEAR INFLUENCE COEFFICIENTS	
R''_{jk}	$-\alpha\frac{b_k}{2}$
R''_{kk}	$-\alpha\frac{b_k}{2}$
DISPLACEMENT LOAD COEFFICIENT	
$\Sigma \bar{V}_k^*$	$+\frac{\cos\phi_\ell}{\sin\gamma_k}\bar{P}_k$

DISPLACEMENT EQUATIONS

$$\Sigma R_j \bar{v}_j = + R_{kj}\bar{v}_k + R'_{jj}\bar{m}_j + R'_{kj}\bar{m}_k + R''_{jj}\bar{T}_j + \Sigma \bar{V}_j^*$$

$$\Sigma R_k \bar{v}_k = + R_{jk}\bar{v}_j + R'_{jk}\bar{m}_j + R'_{kk}\bar{m}_k + R'_{\ell k}\bar{m}_\ell + R''_{jk}\bar{T}_j + R''_{kk}\bar{T}_k + \Sigma \bar{V}_k^*$$

CARRY-OVER EQUATIONS

$$\bar{v}_j = + r_{kj}\bar{v}_k + r'_{jj}\bar{m}_j + r'_{kj}\bar{m}_k + r''_{jj}\bar{T}_j + \bar{v}_j^*$$

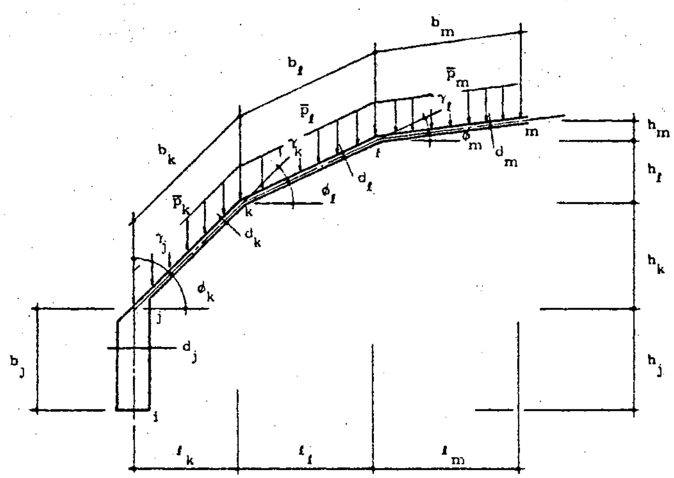
$$\bar{v}_k = + r_{jk}\bar{v}_j + r'_{jk}\bar{m}_j + r'_{kk}\bar{m}_k + r'_{\ell k}\bar{m}_\ell + r''_{jk}\bar{T}_j + r''_{kk}\bar{T}_k + \bar{v}_k^*$$

EQUIVALENTS

$$r_{kj} = + \frac{R_{kj}}{\Sigma R_j} \qquad r'_{kj} = + \frac{R'_{kj}}{\Sigma R_j} \qquad \bar{v}_j^* = + \frac{\Sigma \bar{V}_j^*}{\Sigma R_j} \qquad r'_{jk} = + \frac{R'_{jk}}{\Sigma R_k} \qquad r'_{\ell k} = + \frac{R'_{\ell k}}{\Sigma R_k} \qquad r''_{kk} = + \frac{R''_{kk}}{\Sigma R_k}$$

$$r'_{jj} = + \frac{R'_{jj}}{\Sigma R_j} \qquad r''_{jj} = + \frac{R''_{jj}}{\Sigma R_j} \qquad r_{kj} = + \frac{R_{kj}}{\Sigma R_k} \qquad r'_{kk} = + \frac{R'_{kk}}{\Sigma R_k} \qquad r''_{jk} = + \frac{R''_{jk}}{\Sigma R_k} \qquad \bar{v}_k^* = + \frac{\Sigma \bar{V}_k^*}{\Sigma R_k}$$

TABLE 10	MOMENT FUNCTIONS	EDGE BEAM



TRANSVERSE SECTION

MOMENT INFLUENCE COEFFICIENTS	
ΣF_j	$+\dfrac{1}{\alpha^2 J_j G} + \dfrac{b_k}{3EI'_k}$
G_{kj}	$+\dfrac{b_k}{6EI'_k}$
DISPLACEMENT INFLUENCE COEFFICIENTS	
G'_{jj}	$-EI_{jy}\alpha^4\left(\dfrac{1}{JG\alpha^2 + \dfrac{b_j^2}{2}\ EI_{jy}\alpha^4}\right.$
G'_{kj}	$+EI_{jy}\alpha^4\left(\dfrac{1}{JG\alpha^2 + \dfrac{b_j^2}{2}\ EI_{jy}\alpha^4}\right.$
$G'_{\ell j}$	$-\dfrac{1}{b_k}\left(\dfrac{1}{\sin\gamma_k}\right)$
MOMENT LOAD COEFFICIENT	
$\Sigma\bar{\tau}^*_j$	$+\dfrac{\bar{p}_k b_k^2 \ell_k}{24EI'_k}$

THREE MOMENT EQUATION

$$O = \Sigma F_j \bar{m}_j + G_{kj}\bar{m}_k + G'_{jj}\bar{v}_j + G'_{kj}\bar{v}_k + G'_{\ell j}\bar{v}_\ell + \Sigma\bar{\tau}^*_j$$

CARRY-OVER EQUATION

$$\bar{m}_j = + g_{kj}\bar{m}_k + g'_{jj}\bar{v}_j + g'_{kj}\bar{v}_k + g'_{\ell j}\bar{v}_\ell + \bar{m}^*_j$$

EQUIVALENTS

$$g_{kj} = -\frac{G_{kj}}{\Sigma F_j} \qquad g'_{jj} = -\frac{G'_{jj}}{\Sigma F_j} \qquad g'_{kj} = -\frac{G'_{kj}}{\Sigma F_j}$$

$$g'_{\ell j} = -\frac{G'_{\ell j}}{\Sigma F_j} \qquad \bar{m}^*_j = -\frac{\Sigma\bar{\tau}^*_j}{\Sigma F_j}$$

VITA

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Candidate for the Degree of

Master of Science

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